A Technical report on the Design and Simulink Implementation of

Multi Tone Code Division Multiple Access

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History of OFDM:

The idea of multi carrier transmission goes back to the 1960s (Chang 1966; Chang and Gibby 1968; Saltzberg 1967). The original idea was indeed a physical realization of the concept of multi carrier transmission by using a large number of oscillators. The idea to simplify the implementation by using Fourier transform techniques goes back to (Weinstein and Ebert 1971) and was further developed by Hirosaki (1981). For a long time, however, the implementation of multi carrier transmission by digital circuits for high-speed data communication was still out of question. Thus, these fundamental ideas were widely unknown not only for practical engineers but even for the scientific community.

It was pointed out by Cimini (1985) that OFDM with guard interval is especially suited for the mobile radio channel. This paper seems to be an inspiration for people at the French telecommunication and broadcasting research institute, CCETT, to propose OFDM as a digital broadcasting transmission system for mobile receivers (Alard and Lassalle 1987). It was the merit of these engineers to recognize that the time of OFDM had come and its realization by digital circuits had become a distinct possibility. In the European Digital Audio Broadcasting project, this system proposal became a very serious candidate and, at the end of the project, an OFDM system was standardized in 1993 (see EN300401 2001a for a recent update of the standard). An exhaustive treatment of the DAB system that is also very helpful for the practical engineer can be found in (Hoeg and Lauterbach 2003). A comprehensive overview about multi carrier modulation and its history can be found in (Bingham 1990) and in (Gitlin *et al.* 1993).

The concept of multi carrier transmission:

Let us consider a digital transmission scheme with linear carrier modulation (e.g. *M*-PSK or *M*-QAM) and a symbol duration denoted by T_S. Let B be the occupied bandwidth.
Typically, B is of the order of T_S^{-1}, for example, $B = (1 + \alpha)\, T_S^{-1}$ for raised-cosine pulses with rolloff factor α. For a transmission channel with a delay spread τ_m, a reception free of intersymbol interference (ISI) is only possible if the condition $\tau_m << T_S$ is fulfilled. As a consequence, the possible bit rate $R_b = \log_2 (M)\, T_S^{-1}$ for a given single carrier modulation scheme is limited by the delay spread of the channel.

The simple idea of multi carrier transmission to overcome this limitation is to split the data stream into K substreams of lower data rate and to transmit these data substreams on adjacent *subcarriers*, as depicted in Figure 1 for $K = 8$. This can be regarded as a transmission parallel in the frequency domain, and it does not affect the total bandwidth that is needed. Each subcarrier has a bandwidth B/K, while the symbol duration T_S is increased by a factor of K, which allows for a K times higher data rate for a given delay spread. The factor K, however, cannot be increased arbitrarily, because too long symbol durations make the transmission too sensitive against the time incoherence of the channel that is related to the maximum Doppler frequency v_{max}.

There, the condition $v_{max}\, T_S << 1$ must be fulfilled. Both conditions can only be valid simultaneously if the coherency factor $\kappa = v_{max}\, \tau_m$ fulfills the condition $\kappa << 1$. For a given and sufficiently small factor κ, one should expect that there exists a symbol duration T_S that satisfies both requirements together to give the best possible transmission conditions for that channel. Then choose this optimal symbol duration that is matched to the channel and parallelize the given data stream in an appropriate way.

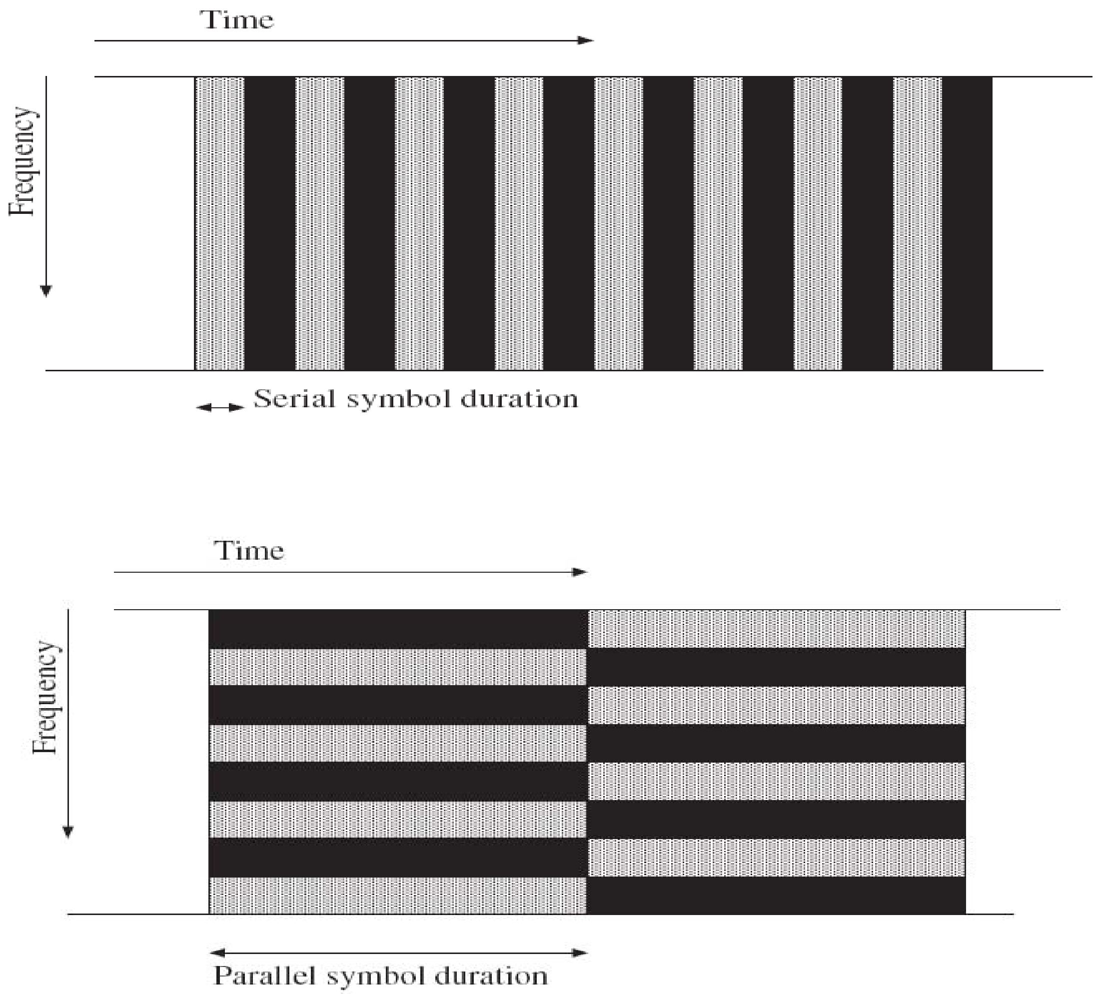


Diagram 1: Concept of Multi carrier transmission

There are two possible ways to look at (and to implement) this idea of multi carrier transmission. Both are equivalent with respect to their transmission properties. Even though mathematically closely related, they differ slightly from the conceptual point of view.

The first one emphasizes the multi carrier concept by having K individual carriers that are modulated independently. This concept is the favorite textbook point of view. The second one is based on a filter bank of K adjacent bandpass filters that are excited by a parallel data stream, leading to a transmission parallel in frequency. This concept is usually implemented in practical systems.
The first concept keeps the subcarrier frequency fixed and considers the modulation in time domain for each subcarrier. The second one keeps a time slot of length T_S fixed and considers modulation in frequency domain for each time slot. In the first setup, the data stream is split up into K parallel substreams and each one is modulated on its own subcarrier at frequency f_k in the complex baseband, described by the complex harmonic wave exp $(j2\pi f_k t)$. We denote the complex (e.g. PSK or QAM) modulation symbols by s_{kl}, where k is the frequency index and l is the time index. With a baseband transmission pulse $g(t)$, this setup can be visualized by Figure 2: The parallel data stream excites replicas of the same pulse-shaping filter $g(t)$ and the filtered signals are modulated on the different carriers and summed up before transmission. The complex baseband signal is then given by the expression

$$s(t) = \sum_k e^{j2\pi f_k t} \sum_l s_{kl} g(t - lT_S),$$

where T_S is the parallel symbol duration. If it is convenient, the time index l may run from zero or minus infinity to plus infinity. Since every real transmission starts and stops at some time instant, it is more realistic to let l run from 0 to $L - 1$, where L is an integer. The frequency index may only run over a limited domain of, say, K different frequencies. From the mathematical point of view, choose $k = 0, 1 \ldots, K - 1$. The engineer, however, would prefer to have f_0 in the middle corresponding to DC in the complex baseband and to the center frequency f_c in the passband, with negative k for the lower sideband and positive k for the upper sideband. For reasons of symmetry, we may then choose the number of carriers to

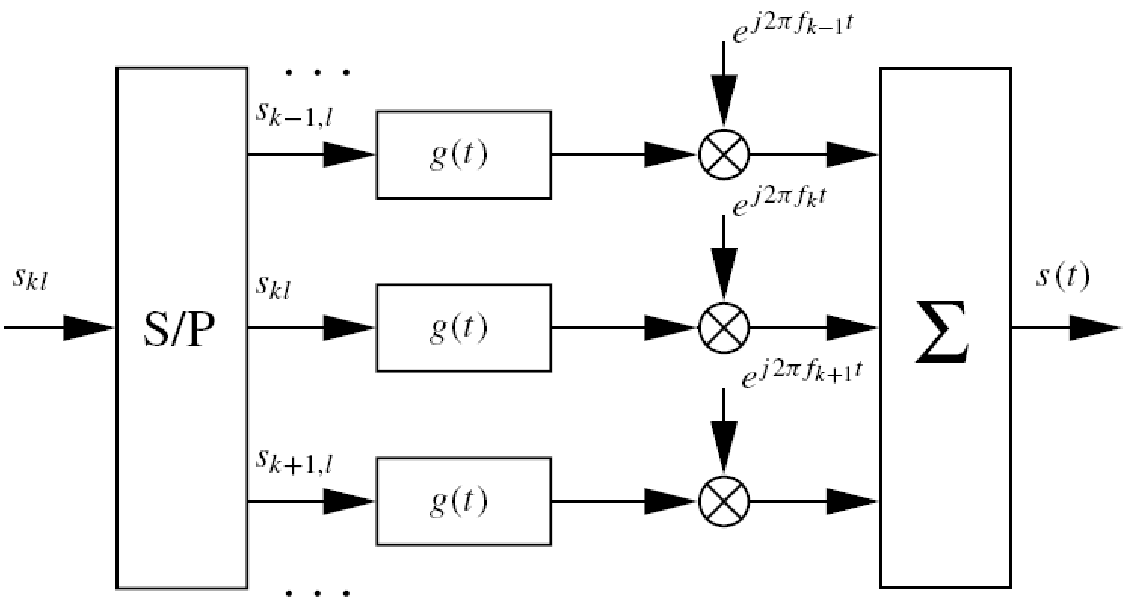


Diagram 2: Multi carrier modulation by classical approach

be $K + 1$, where K is an even integer, and let $k = 0, \pm 1, \pm 2, \ldots, \pm K/2$. The passband signal is then given by

$$\tilde{s}(t) = \Re\left\{\sqrt{2}\, e^{j2\pi f_c t} s(t)\right\} = \Re\left\{\sqrt{2} \sum_k e^{j2\pi (f_c + f_k)t} \sum_l s_{kl} g(t - lT_S)\right\}.$$

For reasons due to implementation, in practical systems, the DC component will sometimes be left empty, that is only the subcarriers at $k = \pm 1, \pm 2, \ldots, \pm K/2$ are used.

In the second setup, from a base transmit pulse $g(t)$ frequency-shifted replicas of this pulse are obtained as

$$g_k(t) = e^{j2\pi f_k t} g(t),$$

that is, if $g(t) = g_0(t)$ is located at the frequency $f = 0$, then $g_k(t)$ is located at $f = f_k$.
In contrast to the first scheme, for each time instant l, the set of K (or $K + 1$) modulation symbols is transmitted by using *different* pulse shapes $g_k(t)$: the parallel data stream excites a filter bank of K (or $K + 1$) different bandpass filters. The filter outputs are then summed up before transmission. This setup is depicted in Figure 3. The transmit signal in the complex baseband representation is given by

$$s(t) = \sum_l \sum_k s_{kl} g_k(t - lT_S).$$

For the domain of the summation indices k and l, the same remarks apply as for the discussion of the first setup. Define

$$g_{kl}(t) = g_k(t - lT_S) = \mathrm{e}^{j2\pi f_k(t-lT_S)} g(t - lT_S)$$

to get the compact expression

$$s(t) = \sum_{kl} s_{kl} g_{kl}(t).$$

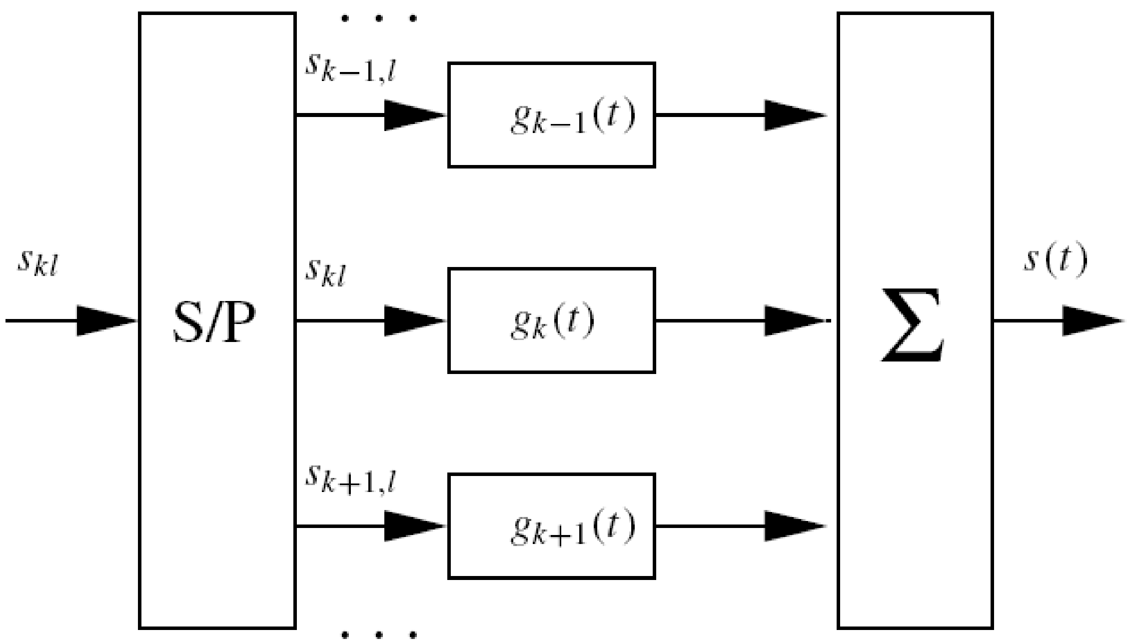


Diagram 3: Multi carrier modulation by Filter bank approach

It is obvious that the first setup is back if the modulation symbols s_{kl} are replaced by $s_{kl}\exp(-j2\pi f_k l T_S)$ in the Equation. Such a time-frequency-dependent phase rotation does not change the performance, so both methods can be regarded as equivalent. However, the second - the filter bank - point of view is closer to implementation, especially for the case of OFDM, where the filter bank is just an FFT.

Implementation by FFT:

The narrow-sense OFDM with the Fourier base is very simple to implement. Consider one time interval (e.g. that for l = 0), the transmit signal is given by

$$s(t) = \frac{1}{\sqrt{T}} \sum_{k=-K/2}^{K/2} s_k \exp\left(j2\pi\frac{k}{T}t\right) \Pi\left(\frac{t}{T} - \frac{1}{2}\right).$$

This means that, for each time interval of length T, OFDM is just a Fourier synthesis for that period. The perfectly synchronized receiver just performs a Fourier analysis to recover the data symbols s_k from the signal:

$$s_k = \langle g_k, s\rangle = \frac{1}{\sqrt{T}} \int_0^T \exp\left(-j2\pi\frac{k}{T}t\right) s(t)\,\mathrm{d}t.$$

A Fourier analysis is preferably implemented by means of a fast Fourier transform (FFT), a synthesis by the inverse fast Fourier transform (IFFT), leading to a setup as depicted in Figure 4. The stream of digitally modulated symbols s_{kl} is divided into blocks of length K (or K + 1), discretely Fourier transformed

by the IFFT, digital–analog converted and then transmitted. The FFT length N_{FFT} must be chosen to be significantly larger than K to ensure that the edge effects are negligible at half the sampling frequency and to ensure that the shape of the reconstruction filter of the DAC (digital-to-analog converter) does not affect the significant part of the spectrum. Furthermore, the alias spectra must be suppressed.
To give a concrete example, in the European DAB (Digital Audio Broadcasting) and in the DVB-T (Digital Video Broadcasting-Terrestrial) system (EN300401 2001a; EN300744 2001b; Hoeg and Lauterbach 2003), an FFT with $N_{FFT} = 2048$ is used (among other FFT modes) and the number of modulated carriers is of the order $K \approx 1500$ and $K \approx 1700$, respectively. The $N_{FFT} - K$ remaining spectral coefficients outside the transmission band are set to zero. At the receiver, the baseband signal will be analog-to-digital converted. Then, for each block of N_{FFT} samples, an FFT of that length is performed and the K useful coefficients will be extracted from the N_{FFT} spectral coefficients.

This picture is very suggestive from a practical point of view and one feels easily inclined to believe that it should work, because every block on the transmit site has its corresponding inverse on the receive site, so all the data should be perfectly recovered if every block works perfectly. Without explaining anything about orthogonality, the picture is also suited to convince a practical engineer that the concept of OFDM should work. However, the concept of orthogonality is only hidden in this picture under the cover of Fourier transform theory.

The ideal OFDM signal is not strictly band limited due to the sinc shapes in the spectrum, while an analog signal can only be perfectly represented by its samples if it is strictly band limited. However, the problem of aliasing is a familiar one that occurs in many communications systems. For OFDM transmission, special care must be taken because of the poor spectral decay.

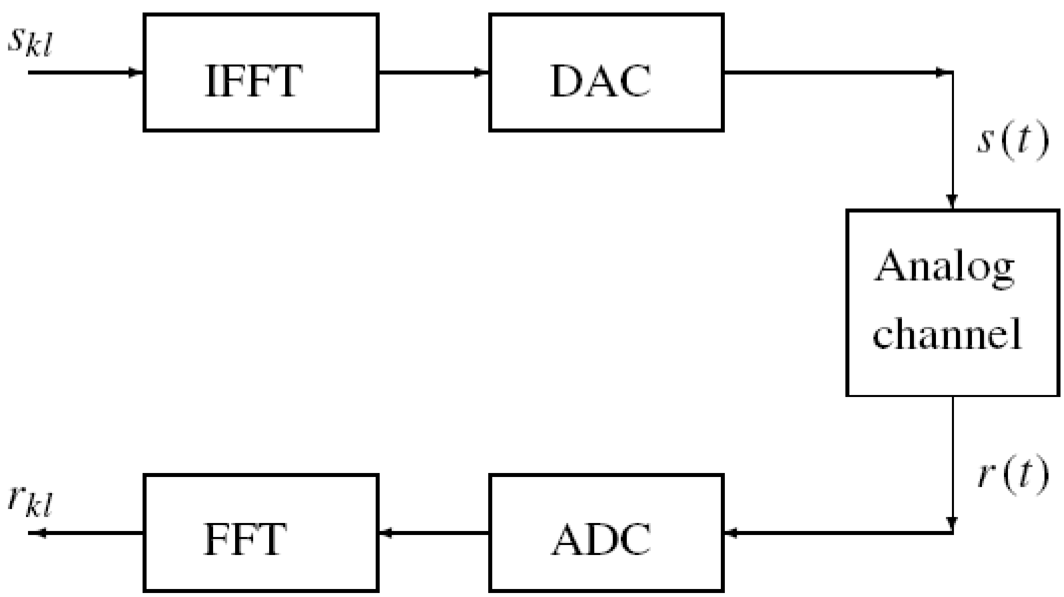


Diagram 4: Multi carrier modulation by IFFT & FFT pair

OFDM Modem Structure:
The principle of any Frequency Division Multiplexing (FDM) system is to split the information to be transmitted into N parallel streams, each of which modulates a carrier using an arbitrary modulation technique. The frequency spacing between adjacent carriers is Δf, resulting in a total signal bandwidth of $N\,\Delta f$. The resulting N modulated and multiplexed signals are transmitted over the channel and at the receiver, parallel receiver branches recover the information. A multiplexer then recombines the N parallel information streams into a high-rate serial stream.

The conceptually simplest implementation of an FDM modem is to employ *N* independent transmitter - receiver pairs, which is often prohibitive in terms of complexity and cost. Weinstein suggested the digital implementation of FDM subcarrier modulators & demodulators based on the Discrete Fourier Transform (DFT).The DFT and its more efficient implementation, the Fast Fourier Transform (FFT) are employed for the baseband OFDM modulation/demodulation process, as it can be seen in the schematic shown in Figure 5. The associated harmonically related frequencies can hence be used as the set of subchannel carriers required by the OFDM system. However, instead of carrying out the modulation / demodulation on a subcarrier by subcarrier basis, as in Hirosaki's early proposal for example, all OFDM subchannels are modulated / demodulated in a single inverse DFT (IDFT) / DFT step.

The serial data stream is mapped to data symbols with a symbol rate of l/T_s employing a general phase and amplitude modulation scheme and the resulting symbol stream is demultiplexed into a vector of *N* data symbols. The parallel data symbol rate is $1/NT_s$ i.e. the parallel symbol duration is *N* times longer than the serial symbol duration T_s. Hence the effects of the dispersive channel - which are imposed on the transmitted signal as the convolution of the signal with the CIR - become less damaging, affecting only a fraction of the extended signaling pulse duration. The inverse FFT (IFFT) of the data symbol vector is computed and the coefficients S_0 to S_{N-1} constitute an OFDM symbol, as seen in the Figure 5.

Since the harmonically related and modulated individual OFDM subcarriers can be conveniently visualized as the spectrum of the signal to be transmitted, it is the IFFT - rather than the FFT - which is invoked, in order to transform the signal's spectrum to the time-domain for transmission over the channel. The associated modulated signal samples **s** are the time domain samples of the OFDM symbol and are transmitted sequentially over the channel at a sample rate of l/T. At the receiver, a spectral decomposition of the received time-domain samples **r** is computed employing an N-tap FFT and the recovered data symbols **R** are restored in serial order and demultiplexed, as seen in Figure 5.

The underlying assumption in the context of OFDM upon invoking the IFFT for modulation is that although *N* frequency-domain samples produce *N* time-domain samples, both signals are assumed to be periodically repeated over an infinite time-domain and frequency domain interval, respectively. In practice, however, it is sufficient to repeat the time-domain signal periodically for the duration of the channel's memory, i.e. for a duration that is comparable to the length of the CIR. This is namely the time interval required for the channel's transient response to die down after exciting the channel with a time-domain OFDM symbol.

Once the channel's transient response time has elapsed, its output is constituted by the steady state response constituted by the received time-domain OFDM symbol. In order to ensure that the received time-domain OFDM symbol is demodulated from the channel's steady-state rather than from its transient - response, each time-domain OFDM symbol is extended by the scaled cyclic extension (C Ext. in Figure 5) or guard interval of N_g samples duration, in order to overcome the inter-OFDM symbol interference due to the channel's memory.

The signal samples received during the guard interval are discarded at the receiver and the N-sample received time-domain OFDM symbol is deemed to follow the guard interval of N_g samples duration. The demodulated OFDM symbol is then generated from the remaining N samples upon invoking the FFT. We note, however that since the transmitted time-domain signal was windowed to the finite duration of $N + N_g$ samples, the corresponding transmitted frequency-domain signal is convolved with the sinc-shaped frequency-domain transfer function of the rectangular time-domain window function. As a result of this frequency-domain convolution, the originally pure line-spectrum of the IFFT output generates a sinc-shaped subchannel spectrum centered on each OFDM sub-carrier.

The samples of the cyclic extension are copied from the end of the time-domain OFDM symbol, generating the transmitted time domain signal depicted in Figure 6. At the receiver, the samples of the cyclic extension are discarded. Clearly, the need for a cyclic extension in time dispersive environments reduces the efficiency of OFDM transmissions by a factor of $N / (N + N_g)$. Since the duration N_g of the necessary cyclic extension depends only on the channel's memory, OFDM transmissions employing a high number of carriers *N* are desirable for efficient operation. Typically a guard interval length of not more than 10% of the OFDM symbol's duration is employed.

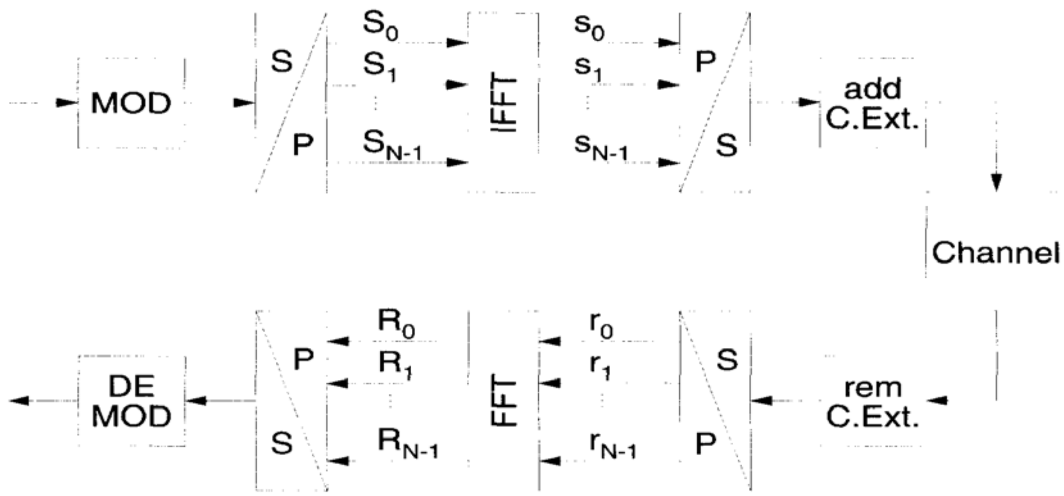


Diagram 5: An OFDM modem

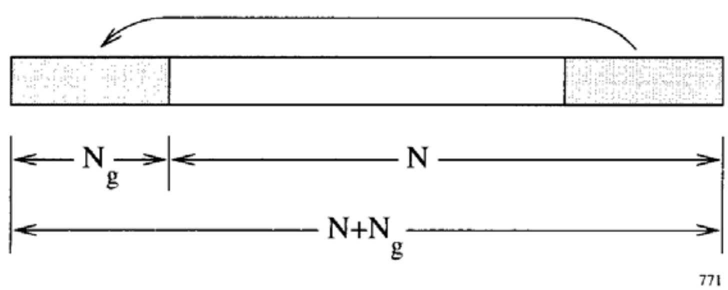


Diagram 6: Adding cyclic prefix to guard against ISI

OFDM systems with convolutional coding and M^2-QAM:
For the OFDM systems, an approach has been chosen which uses standard convolutional coding and QAM modulation with a bit interleaver in between. Such an approach is called *bit interleaved coded modulation* (BICM) in the literature and these systems are probably the first applications of BICM.

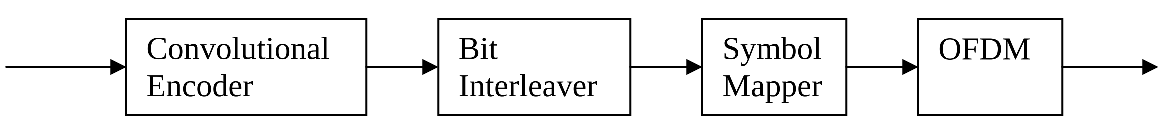


Diagram 7: OFDM with convolutional coding and M^2-QAM

Amalgamating DS CDMA and OFDM:
A DS CDMA system applies spreading sequences in the time domain and uses RAKE receivers to optimally combine the time dispersed energy in order to combat the effects of multipath fading. However in indoor wireless environments the time dispersion is low, on the order of nano seconds and hence a high chip rate, of the order of tens of MHz, is required for resolving the multipath components. This implies a high clock rate, high power consumption as well as implementation difficulties. In order to overcome these difficulties, combining DS-CDMA and multi carrier modulation is an option.

MT-CDMA:
MT-CDMA is a combined technique employing time domain spreading and a multi carrier transmission scheme shown in the Figure 8. The time domain spreading is applied after IFFT stage. The system has a multiple access capability. The main benefit of combining OFDM and DS - spreading is that it is possible to prevent the obliteration of certain subcarriers by deep frequency domain fades. This is achieved by spreading each subcarrier's signal with the aid of a spreading code and thereby increasing the achievable error-resilience, since in case of corrupting a few chips of a spreading code the chances are that the subcarrier signal still may be recovered.

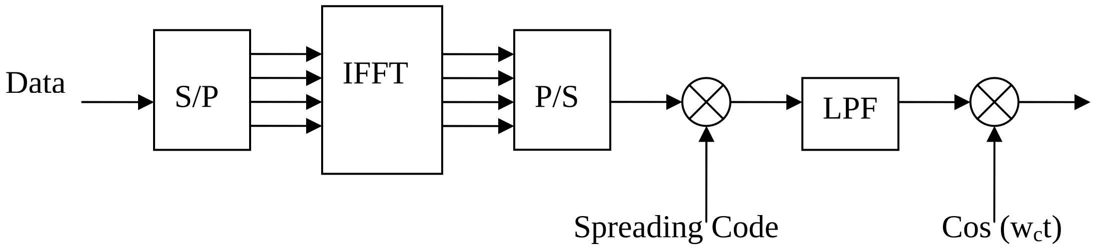
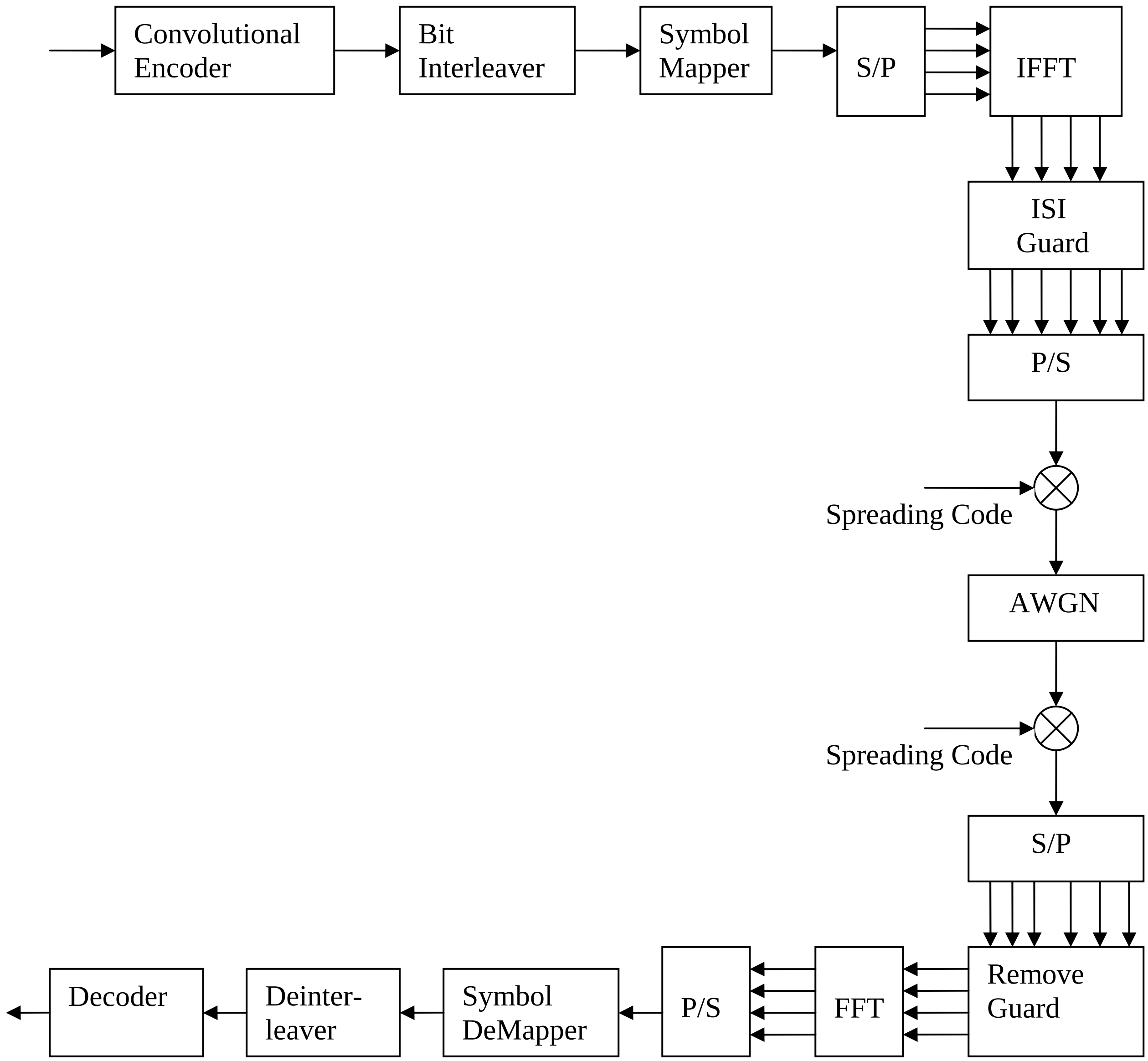
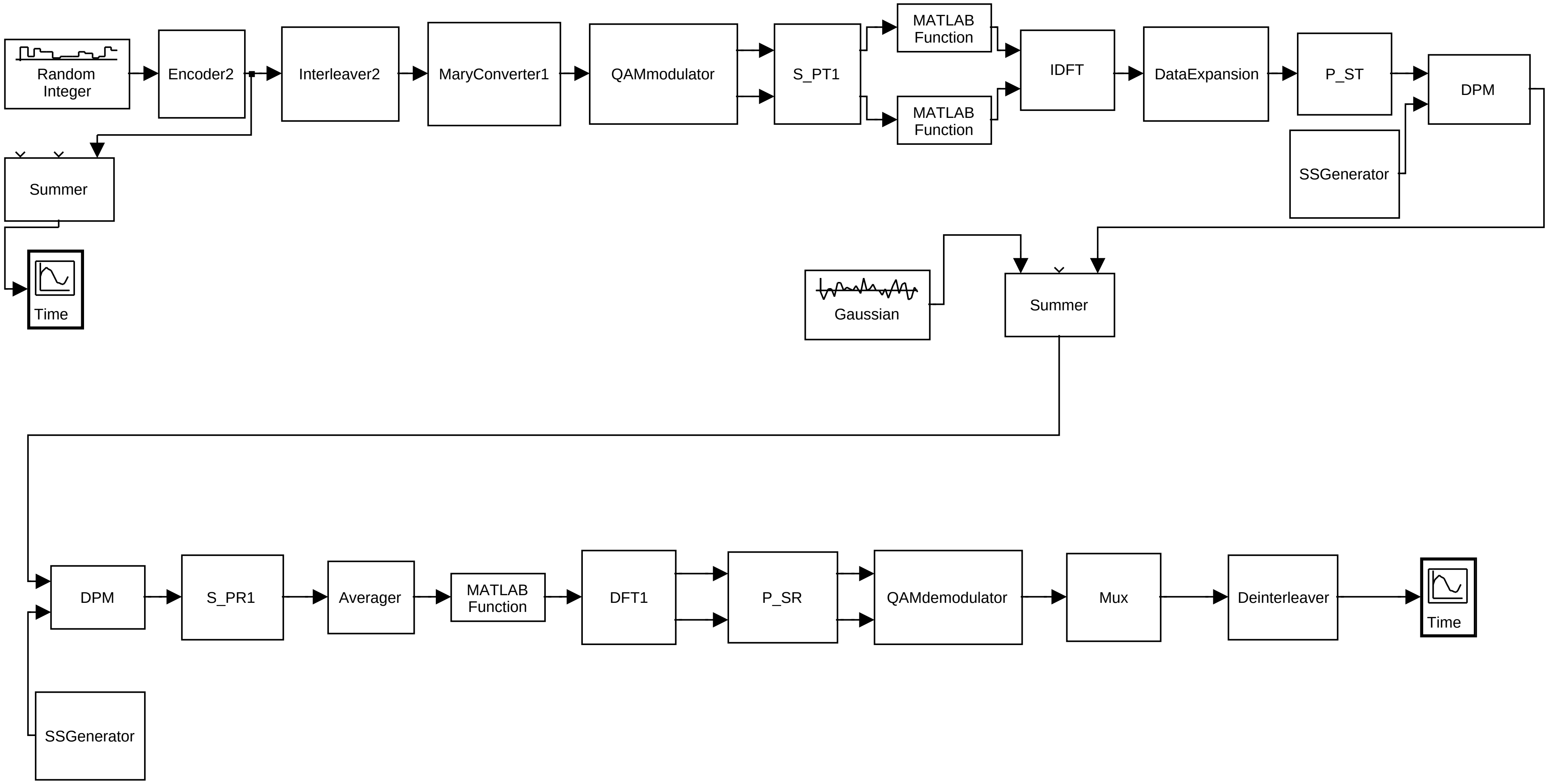


Diagram 8: Transmitter schematic of MT-CDMA

MT-CDMA System in an AWGN Channel:



MT-CDMA System implemented in Simulink:



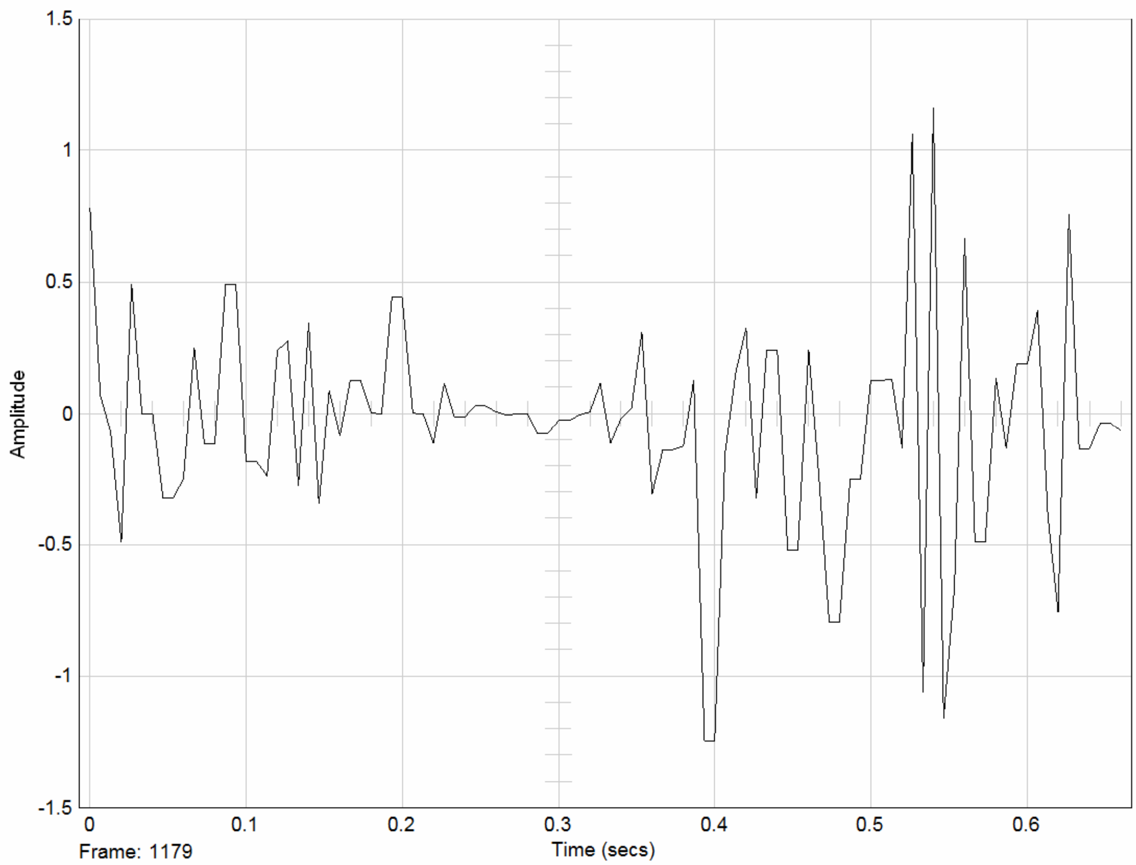
Description of Blocks:

SI	Block Name	Description
1	Random Integer	Random Bit Generator - Simulink Built In Block
2	Encoder 2	Convolutional Encoder Rate 1/3 - Coded Block
3	Interleaver 2	Convolutional Interleaver - Coded Block
4	Mary Converter1	Mapping bits to M-ary symbols - Coded Block
5	QAMmodulator	Mapping M-ary symbols to QAM Inphase & Quadrature symbols - Coded Block
6	S_PT1	Serial to Parallel Tx: Mapping a set of serial QAM Inphase & Quadrature symbols into a parallel complex conjugate symmetric symbol group - Coded Block
7	MATLAB Function - BRO	Function for Bit Reverse Ordering the input to the IDFT block which is a Decimation In Frequency based Radix algorithm. - Coded in MATLAB (not a built-in Matlab function, Simulink does not show the name of a Matlab function included in its model)
8	IDFT	IFFT block for generating the real time domain OFDM signal, based on Radix 2 DIF algorithm - Coded Block.
9	Data Expansion	Duplicates the OFDM samples to guard against ISI in the channel - Coded Block
10	P_ST	Parallel to Serial Tx: Converts the parallel OFDM samples into serial for transmission - Coded Block
11	SSGenerator	Generates the short spreading sequence - Coded Block
12	DPM	Discrete Product Modulator – Spreading the OFDM signal by a user sequence - Coded Block.
13	Gaussian	Gaussian noise generator - Simulink Built In Block
14	S_PR1	Serial to Parallel Rx: Converts the serial OFDM samples into a parallel sample group for detection - Coded Block
15	Averager	Removes the guard against ISI to obtain independent OFDM samples - Coded Block.
16	DFT1	FFT block for converting the real time domain OFDM signal into complex conjugate symmetric symbol group based on Radix 2 Decimation In Time algorithm - Coded Block.
17	P_SR	Parallel to Serial Rx: Mapping a parallel complex conjugate symmetric symbol group into a set of serial QAM Inphase & Quadrature symbols - Coded Block
18	QAMdemodulator	Mapping QAM Inphase & Quadrature symbols to M-ary symbols - Coded Block
19	Mux	Mapping M-ary symbols to bits - Coded Block
20	Deinterleaver	Convolutional Deinterleaver - Coded Block
21	Summer	Block that adds the input signals - Coded Block
22	Time	Block used for data display - Simulink Built In Block

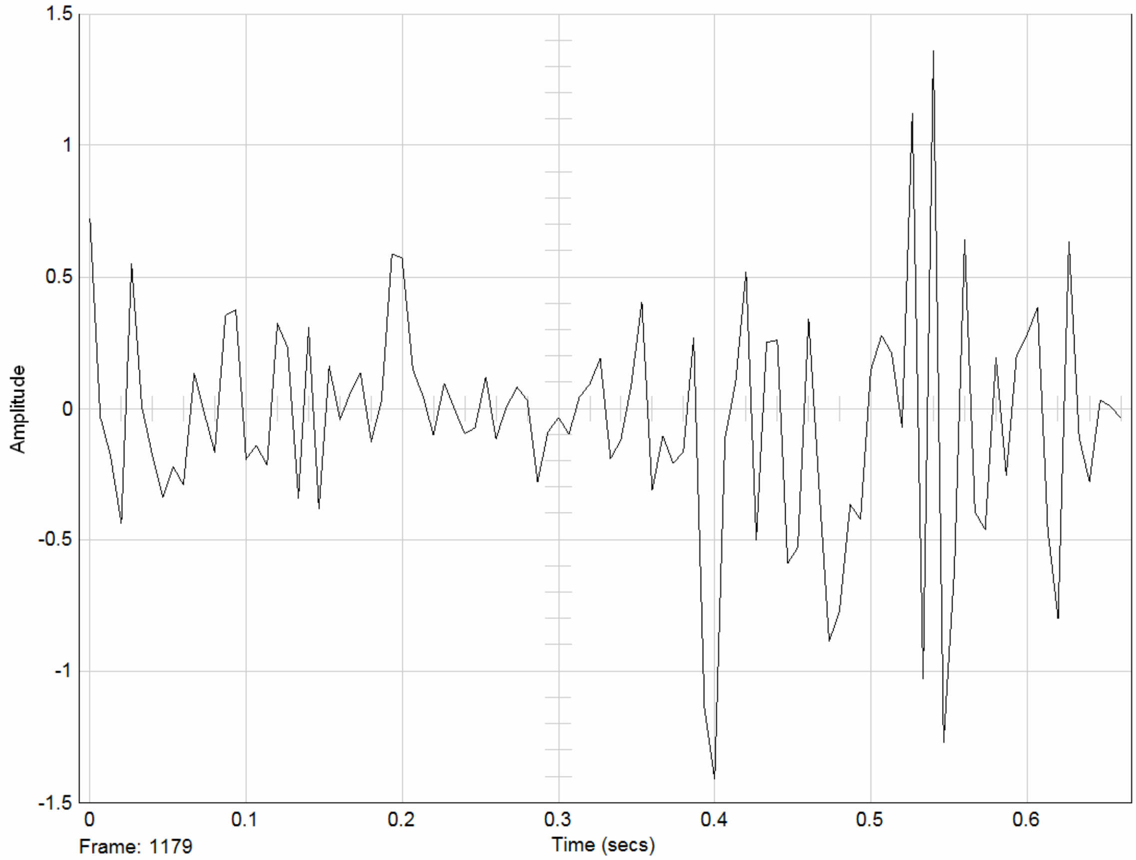
Input Parameters to the Simulink Blocks:

SI	Block Name	Input Parameters	Values
1	Encoder2	1. Input Symbol Duration 2. Symbol Size	1/50 2
2	Interleaver2	Input Symbol Duration	1/150
3	Mary Converter1	1. Bit vector Length 2. Input Symbol Duration	4 1/150
4	QAMmodulator	1. M-ary Number 2. Input Symbol Duration	16 1/150
5	S_PT1	1. M-ary Number 2. Input Symbol Duration 3. Function of Number of subcarriers $(N_{SC} / 2) + 1$	16 1/150 17
6	MATLAB Function - BRO	IFFT/FFT Length	32
7	IDFT	Function of IFFT/FFT Length $(N_{FFT} / 2) + 1$	17
8	Data Expansion	1. Function of Number of subcarriers $(N_{SC} / 2) + 1$ 2. Expanded Length	17 64
9	P_ST	1. M-ary Number 2. Input Symbol Duration 3. Function of Number of subcarriers $(N_{SC} / 2) + 1$ 4. Expanded Length	16 1/150 17 64
10	SSGenerator	Chip duration	1/150
11	DPM	Sample Time	1/150
12	S_PR1	1. M-ary Number 2. Input Symbol Duration 3. Function of Number of subcarriers $(N_{SC} / 2) + 1$ 4. Expanded Length	16 1/150 17 64
13	Averager	1. Function of Number of subcarriers $(N_{SC} / 2) + 1$ 2. Expanded Length	17 64
14	DFT1	1. Function of IFFT/FFT Length $(N_{FFT} / 2) + 1$ 2. M-ary Number	17 16
15	P_SR	1. M-ary Number 2. Input Symbol Duration 3. Function of Number of subcarriers $(N_{SC} / 2) + 1$	16 1/150 17
16	QAMdemodulator	1. M-ary Number 2. Input Symbol Duration	16 1/150
17	Mux	1. Bit vector Length 2. Input Symbol Duration	4 1/150
18	Deinterleaver		
19	Summer	Sample Time	1/150

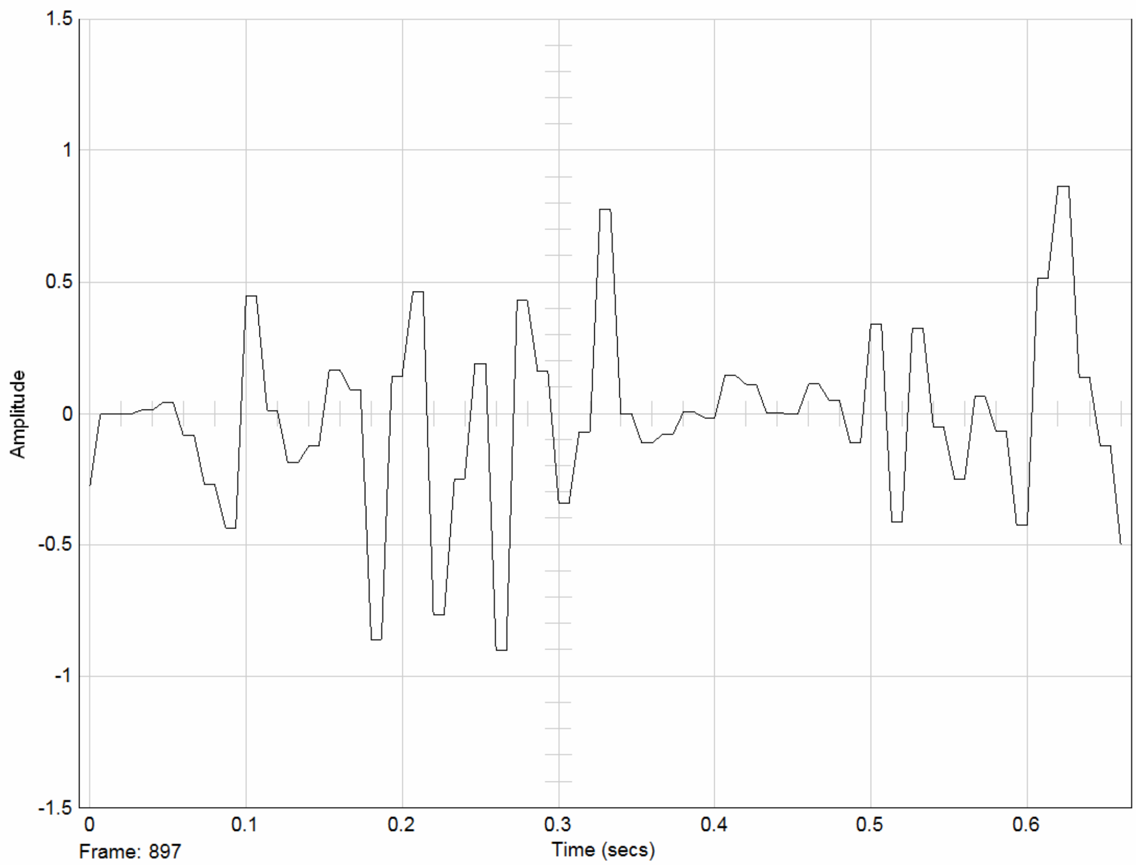
AWGN Channel Input for the MT-CDMA system:



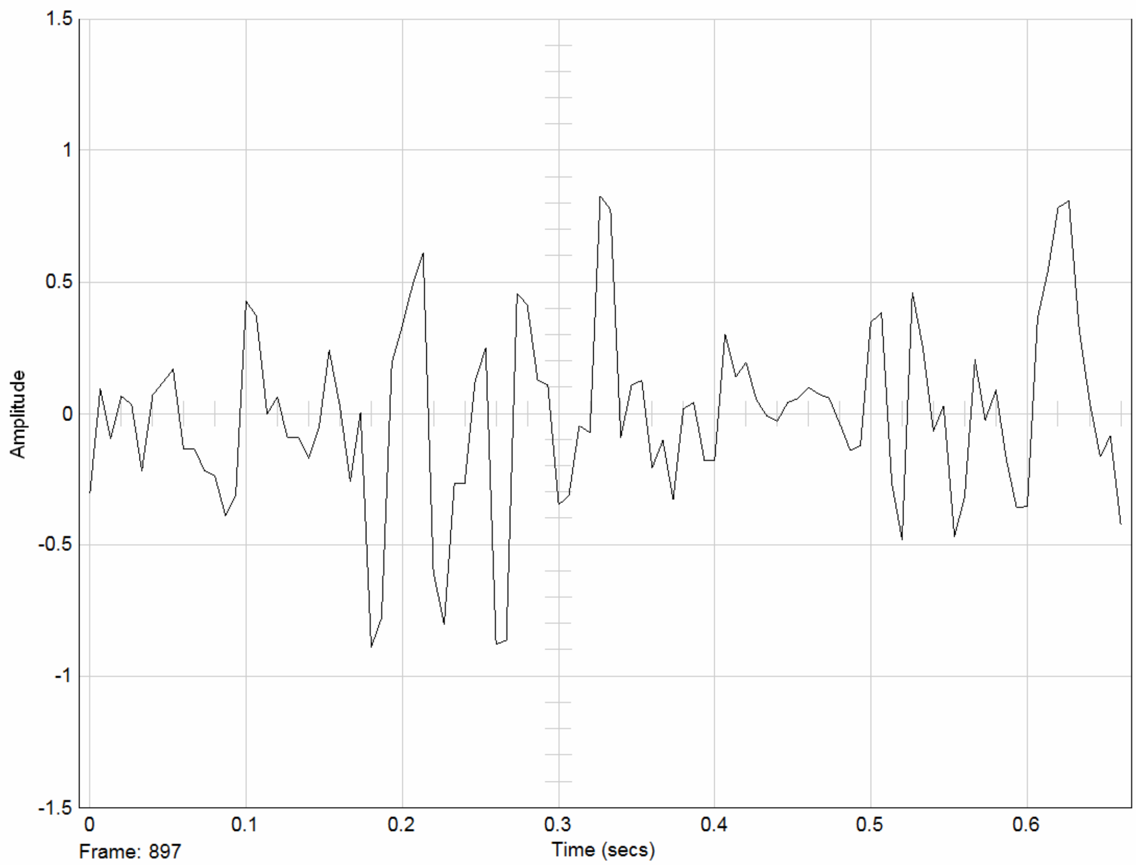
AWGN Channel Output for the MT-CDMA system:



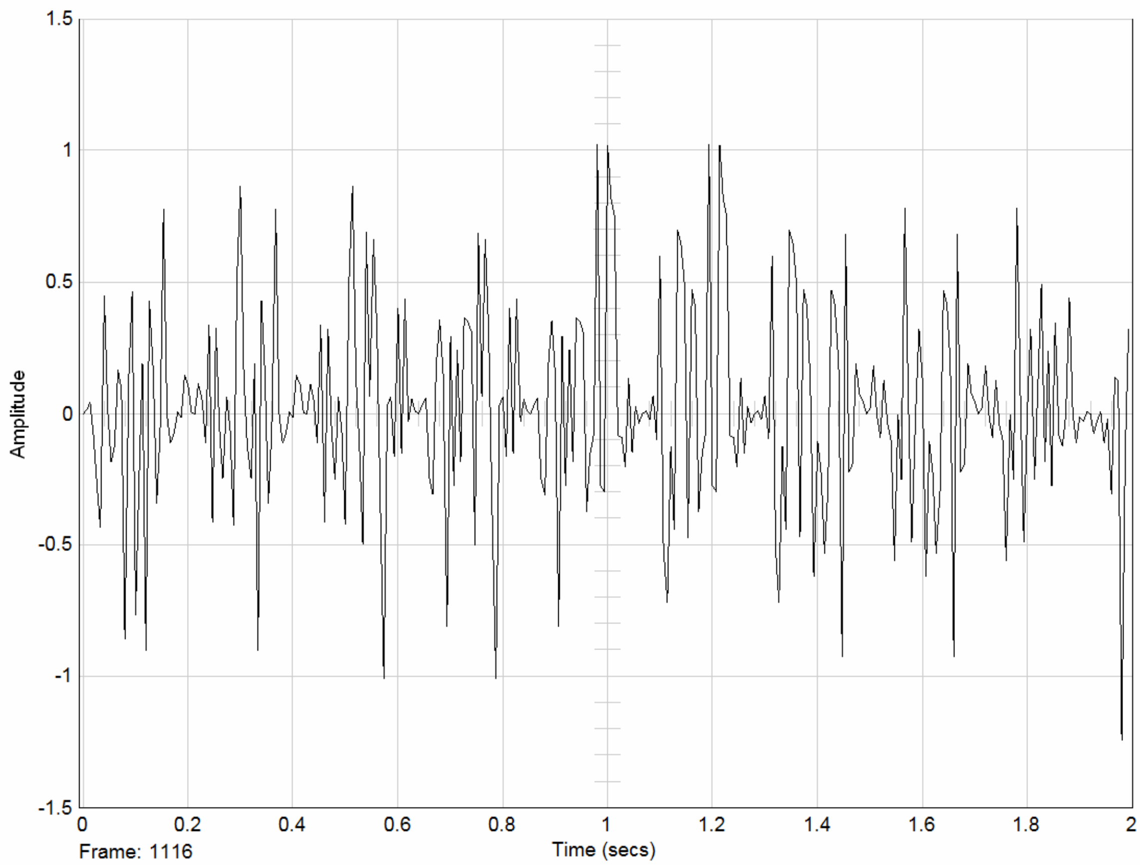
Unspread OFDM Tx signal for the MT-CDMA system:



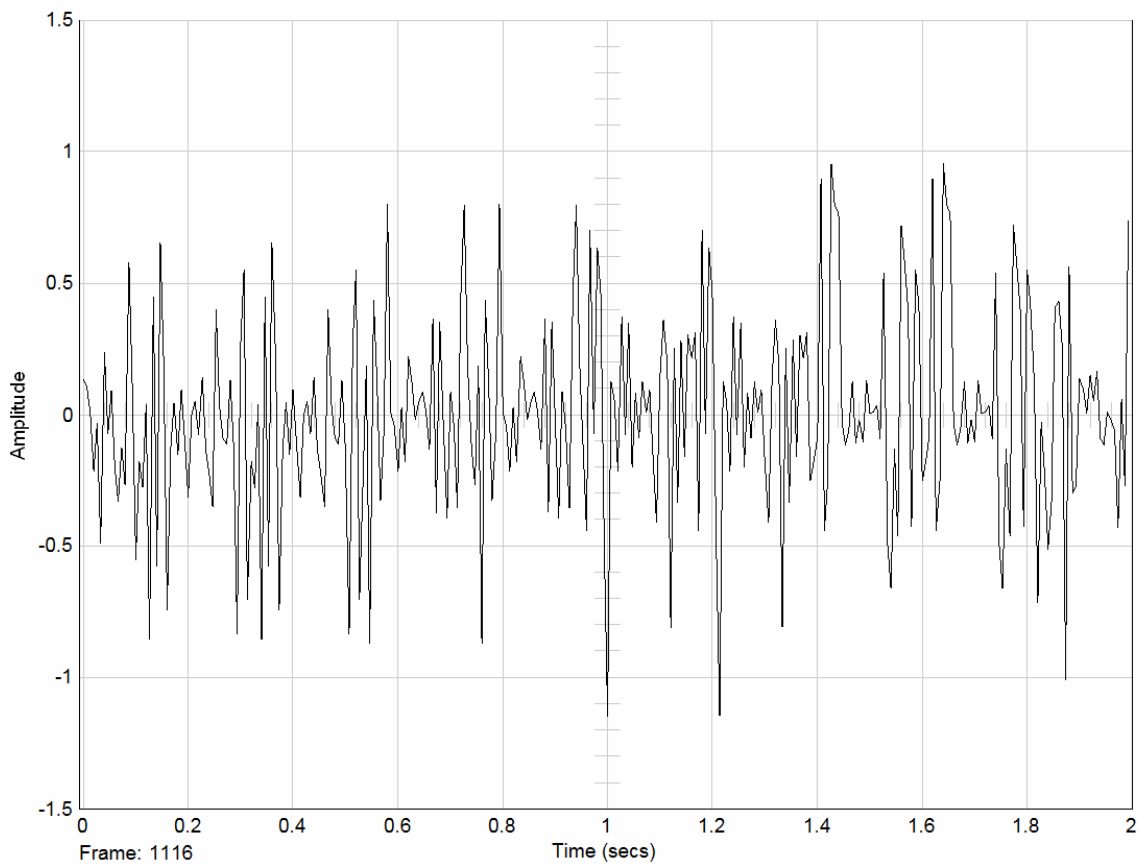
Unspread OFDM Rx signal for the MT-CDMA system:



IDFT Output for the MT-CDMA system:



DFT Input for the MT-CDMA system:
DFT input at the receiver is delayed from the IDFT output at the transmitter by 0.4 secs.



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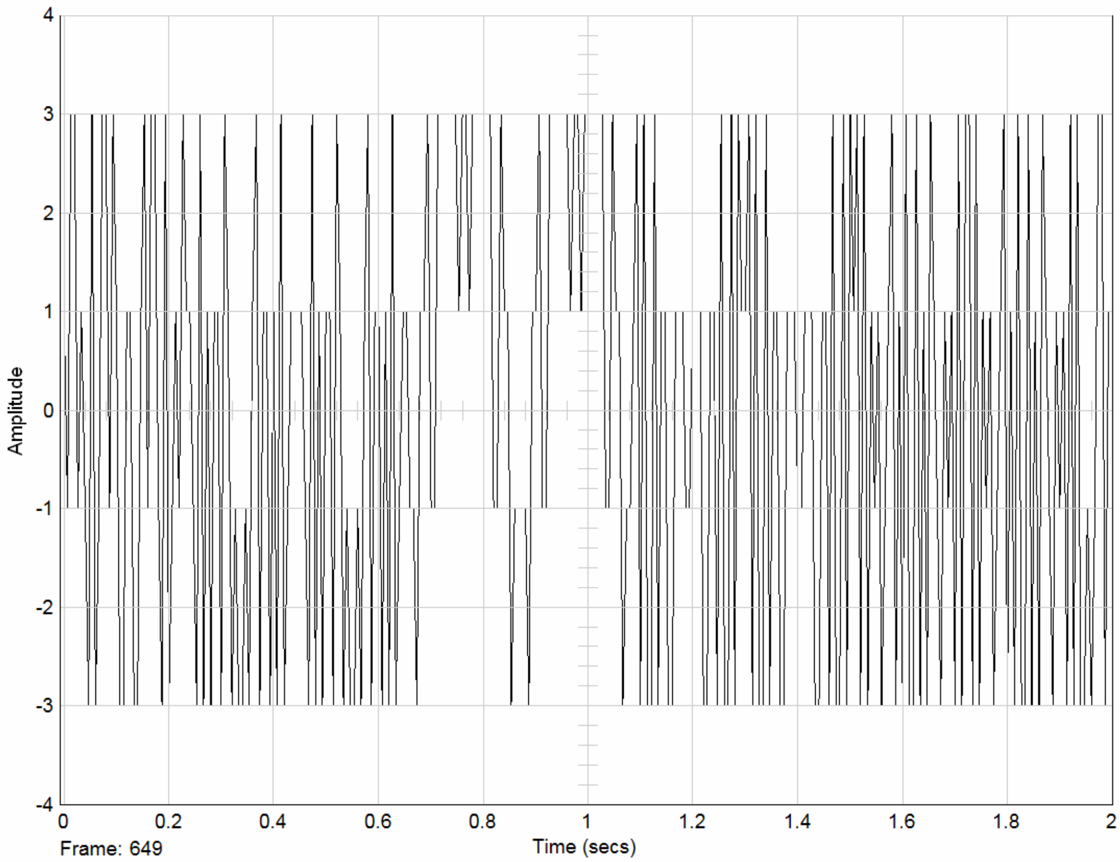
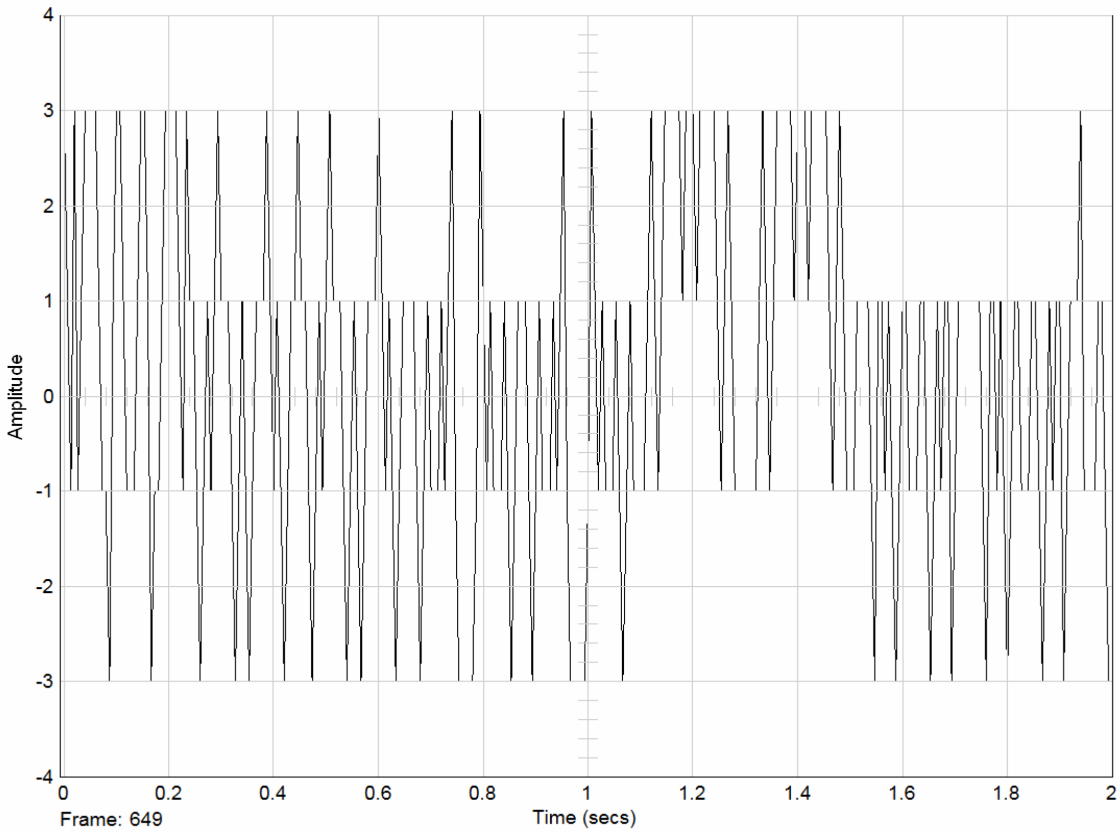
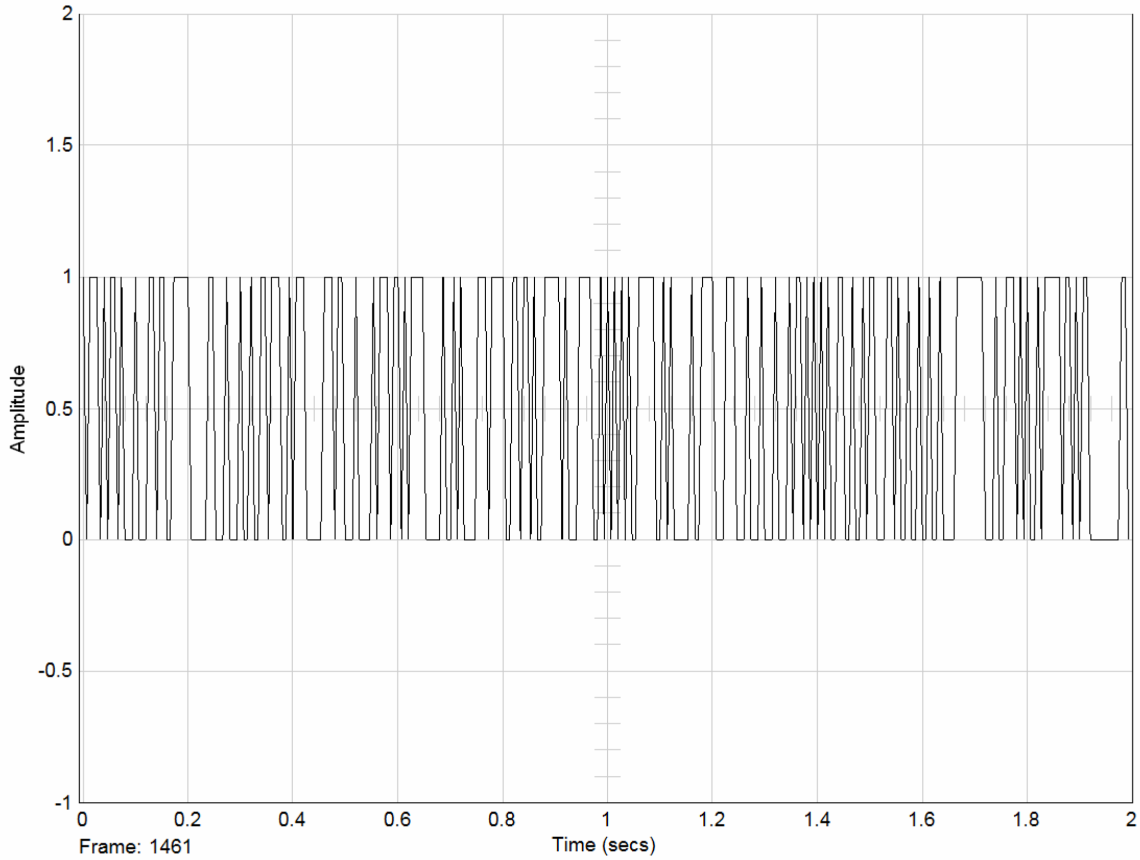
IDFT Real Input for the MT-CDMA system:
Inphase symbols are in the range [-3,-1, 1, 3] for M=16.



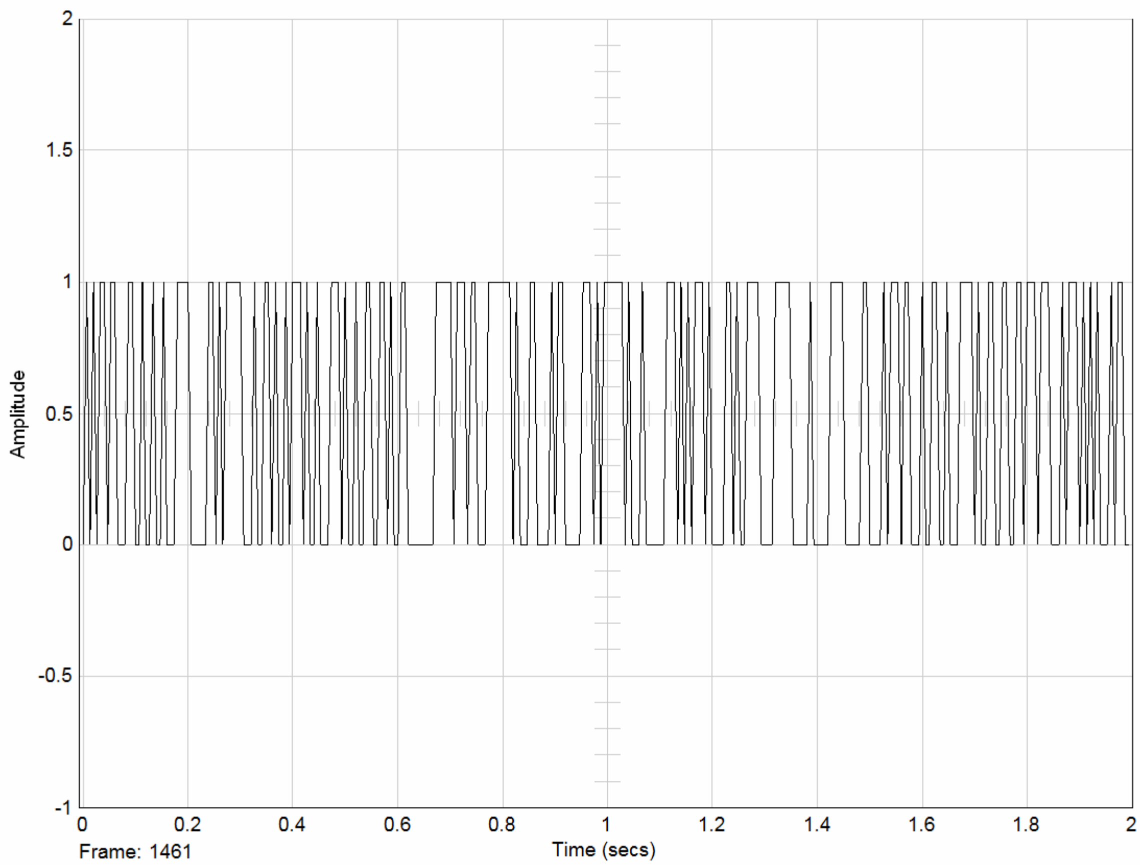
DFT Real Output for the MT-CDMA system:
Inphase symbols are in the range [-3,-1, 1, 3] for M=16.
Inphase symbols at the Rx are delayed from the inphase symbols at the Tx by 0.4 secs. Errors are introduced into the inphase symbols at the Rx due to the Hard decision output by the DFT block.



Interleaver Input for the MT-CDMA system:



Deinterleaver Output for the MT-CDMA system:



Conclusions:
Combining the techniques of OFDM and CDMA, a MT-CDMA system operating in AWGN conditions was studied. Hard decision rule used by the DFT block introduced errors into the inphase and quadrature symbols at the receiver. These errors propagated through the remaining blocks of the receiver i.e Symbol Demapper, Deinterleaver. Use of soft decision in the receiver blocks will improve the symbol error rate. AWGN is too simple a channel to be operating in, so more hostile practical wireless channels need to be considered in future work. Use of the Turbo / LDPC codes and the appropriate decoding schemes in the MT-CDMA system will greatly improve the performance. Currently in the system pseudo noise sequences are used for the spreading operation, use of sequences like Kasami, Gold sequences can improve the system performance. Currently the system uses Radix 2 based FFT and IFFT algorithms, use of more complicated Radix algorithms for FFT and IFFT may affect the performance.

References:
[1] “OFDM and MC-CDMA for Broadcasting Multi-User Communications, WLANs and Broadcasting “ L. Hanzo, M. Münster, B.J. Choi and T. Keller; Wiley & IEEE Press, July 2003.
[2] “Theory and Applications of OFDM and CDMA: Wideband Wireless Communications” Henrik Schulze and Christian Lueders; John Wiley & Sons, September 23, 2005.
[3] “Digital Communications“ John G Proakis, McGraw Hill, 4 edition, August 15, 2000.