

Verification of Protocol ECMA with Decomposition of Petri Net Model

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ABSTRACT

Decomposition of Petri net model of ECMA communication protocol into minimal functional subnets, left and right communicating systems, subsystems of connection establishing and disconnecting is implemented. A correct protocol ought to be invariant one. Invariance of source model is proved on the base of established invariance of functional subnets. Isomorphism of subnets has allowed the calculation of invariants in the process of consecutive composition of net. Acceleration of computations for decomposition technique was estimated. It is exponential with the respect to net dimension.

Keywords: Protocol, Petri net, Invariant, Functional subnet, Decomposition.

1. INTRODUCTION

Asynchronous character of systems' interaction assuming by standard specifications of telecommunication protocols makes difficult application of traditional methods, aimed to description of allowable sequences of actions. It is caused by that the protocol regulates only rules of interaction, while concrete sequences of actions are various implementations of system behaviour according to protocol. Thus, recently for protocols investigation [1,2] more and more frequently Petri nets [3] are used.

Detailed models of real-life telecommunication protocols represented by Petri nets and constructed on the base of source standard specifications number thousands of elements usually. Such huge dimension creates significant difficulties in application of formal methods for net properties investigation aimed to prove the correctness of protocol.

One of the most powerful and widespread methods of Petri net properties analysis is method of invariants [3]. Implementation of this method consists in solving of linear Diophantine systems of equations over nonnegative integer numbers. Note that, finding of nonnegative integer solutions of linear system is a specific task with especial methods of solution [4,5]. Unfortunately, complexity of these methods is asymptotically exponential. It makes practically impossible invariants calculation for net with more than hundred elements.

The goal of present work is construction of effective methods for telecommunication protocols verification on the base of decomposition technique for Petri net invariants calculation and implementation of these methods for ECMA protocol verification.

In work [6] polynomial algorithm of given Petri net decomposition into minimal functional subnets [6,7] was represented. In work [8] invariants of functional subnets were used for construction of invariants of source net. It was shown that acceleration of computations obtained is exponential with the respect to number of nodes of net.

In present work technique of invariants calculation with consecutive composition of source Petri net out of its minimal functional subnets is studied. The symmetry of systems interaction, characteristic for the majority of telecommunication protocols, is involved.

2. MODEL OF COMMUNICATION PROTOCOL

For case study of Petri net invariants calculation with decomposition the wide known protocol ECMA (European Computer Manufacturer Association) has been chosen. ECMA is transport protocol situated between network and session levels of ISO model. Further, the model of protocol represented in [2] will be used. On the one hand, the model is simplified enough to be studied in article, on another hand, it allow the implementation of decomposition technique. Further studying model represents only connection-disconnection processes and abstracts of the concrete way of data transmission.

Petri net model of protocol ECMA is represented in Fig. 1. Let's remind, that *Petri net* [3] is a triple $N = (P, T, F)$, where $P = \{p\}$ – finite set of nodes named places, $T = \{t\}$ – finite set of nodes named transitions, flow relation $F \subseteq P \times T \cup T \times P$ defines a set of arcs connecting places and transitions. Thus, Petri net is directed bipartite graph; one part of nodes consists of places, another – of transitions. Places are drawn as circles, transitions – as bars. Usually, graph N is supplemented with a marking defining an initial arrangement of tokens in places. Tokens are dynamic elements that move inside net as a result of transitions firing.

A special notation of sets of input and output nodes for places and transitions is introduced:

$$\begin{aligned} \bullet p &= \{t \mid \exists(t, p) \in F\}, & p^\bullet &= \{t \mid \exists(p, t) \in F\}, \\ \bullet t &= \{p \mid \exists(p, t) \in F\}, & t^\bullet &= \{p \mid \exists(t, p) \in F\}. \end{aligned}$$

Three basic parts of model is considered: left interacting system – places $p_1 - p_4$, transitions $t_1 - t_7$; right interacting system – places $p_5 - p_8$, transitions $t_8 - t_{14}$; communication subsystem – places $p_9 - p_{16}$. Semantic description of elements of the model is represented in Table 1.

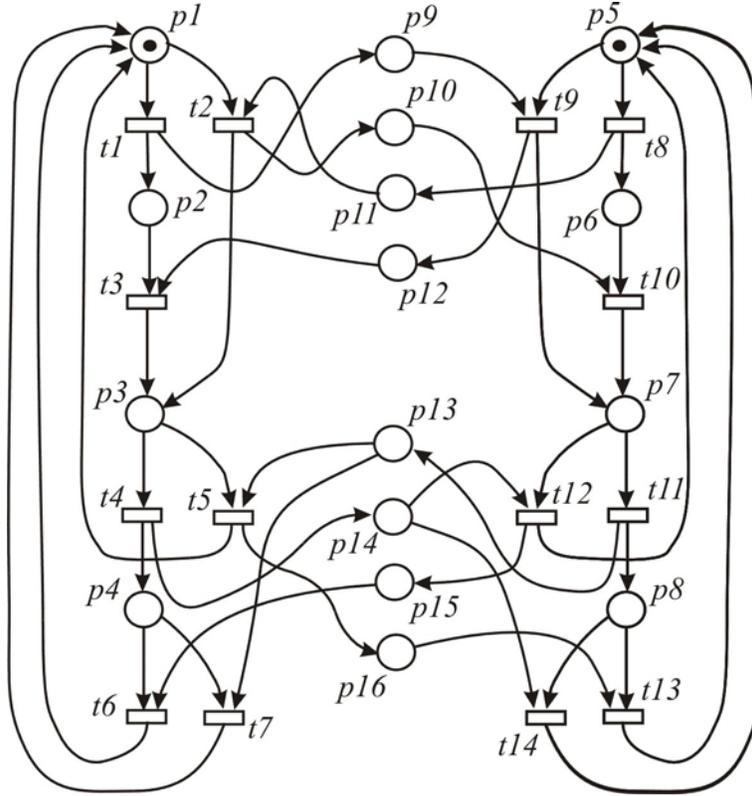


Fig. 1. Model of protocol ECMA

Table 1

Description of model's elements

Place	Description	Transition	Description
p_1, p_5	Initial state of systems	t_1, t_8	Send connection request
p_2, p_6	Waiting of connection	t_2, t_9	Receive connection request
p_3, p_7	Transmission of data	t_3, t_{10}	Receive connection acknowledgement
p_4, p_8	Waiting of disconnection	t_4, t_{11}	Send disconnection request
p_9, p_{11}	Request of connection	t_5, t_{12}	Receive disconnection request
p_{10}, p_{12}	Acknowledgement of connection	t_6, t_{13}	Receive disconnection acknowledgement
p_{13}, p_{14}	Request of disconnection	t_7, t_{14}	Receive counter disconnection request
p_{15}, p_{16}	Acknowledgement of disconnection		

3. DECOMPOSITION OF PROTOCOL

We decompose source model of ECMA protocol represented in Fig. 1 in minimal functional subnets according to decomposition algorithm described in [6].

Let's remind, that functional net [6,7] is a special case of net with input and output places. *Functional net* is a triple $Z = (N, X, Y)$, where N – is Petri net, $X \subseteq P$ – *input places*, $Y \subseteq P$ – *output places*, besides sets of input and output places do not intersect: $X \cap Y = \emptyset$, moreover, input places do not have input arcs, and output places do not have output arcs: $\forall p \in X: \bullet p = \emptyset$, $\forall p \in Y: p^\bullet = \emptyset$.

Places of a set $C = X \cup Y$ are named by *contact* ones, and places of a set $Q = P \setminus (X \cup Y)$ are named by *internal* ones.

Functional net $Z = (N', X, Y)$ is named a *functional subnet* of net N and is denoted as $Z \succ N$ if N' is subnet of N , and, moreover, Z is connected with residuary part of net only through the arcs incidental to either input or output places, besides input places may have only input arcs and output places – only output arcs. Thus

$$\forall p \in X : \{(p, t) | t \in T \setminus T'\} = \emptyset,$$

$$\forall p \in Y : \{(t, p) | t \in T \setminus T'\} = \emptyset,$$

$$\forall p \in Q : \{(p, t) | t \in T \setminus T'\} = \emptyset \wedge \{(t, p) | t \in T \setminus T'\} = \emptyset.$$

Functional subnet is named a *minimal* one, if it does not contain any other functional subnets. According to theorem 2 proved in [6], any functional subnet Z' of a Petri net N is a sun (union) of a finite number of minimal functional subnets. Thus, a set of minimal functional subnets is a generating family for a set of functional subnets of a given Petri net N .

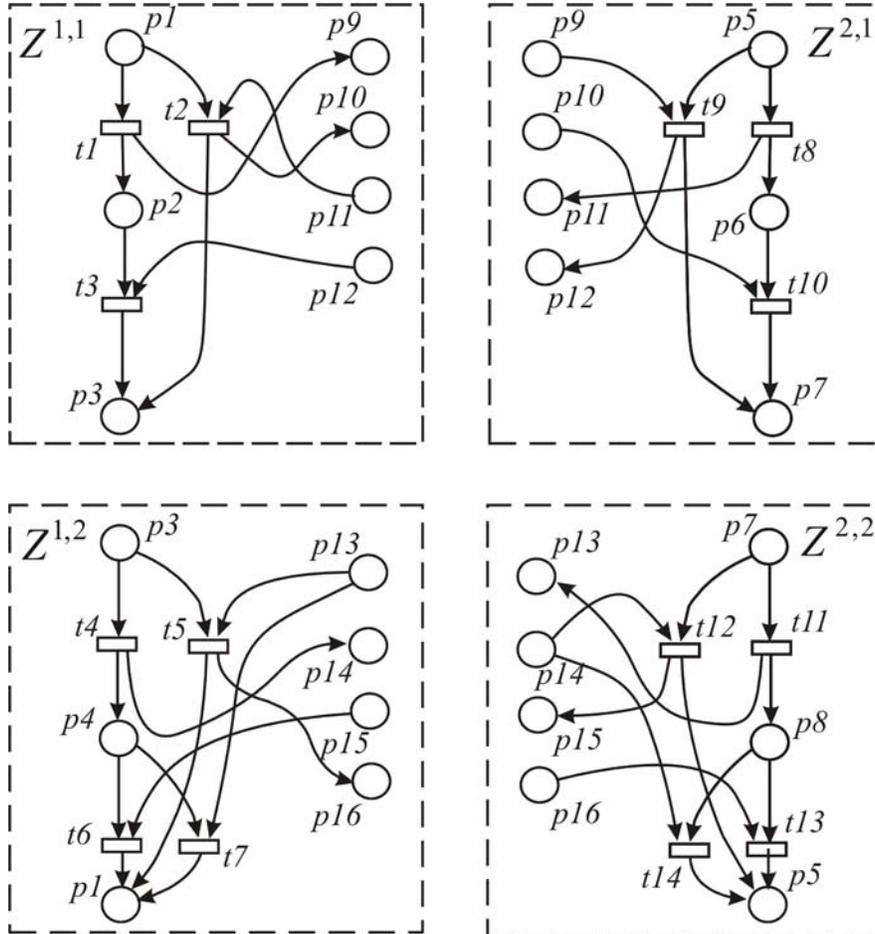


Fig. 2. Decomposition of protocol ECMA

Application of decomposition algorithm to model of ECMA protocol (Fig. 1) results in obtaining of set $\{Z^{1,1}, Z^{1,2}, Z^{2,1}, Z^{2,2}\}$ consisting of four minimal functional subnets represented in Fig. 2. Graph of functional subnets is shown in Fig. 3. Note that, as processes of system interaction are symmetry, so pairs of subnets $Z^{1,1}$ and $Z^{2,1}$, and also $Z^{2,1}$ and $Z^{2,2}$ are isomorphic. Thus, it is necessary to investigate further only properties of two subnets of four obtained.

Different ways of minimal functional subnets composition allow the decomposition of the source model

in left and right interacting systems Z^1, Z^2 and also decomposition in subnets of connection establishing and disconnecting Z^1, Z^2 , where $Z^1 = Z^{1,1} + Z^{1,2}$, $Z^2 = Z^{2,1} + Z^{2,2}$, $Z^1 = Z^{1,1} + Z^{2,1}$, $Z^2 = Z^{2,1} + Z^{2,2}$.

Therefore, decomposition of Petri net model of protocol ECMA into minimal functional subnets was implemented. Moreover, decomposition into left and right interacting systems, subsystems of connection establishing and disconnecting was considered.

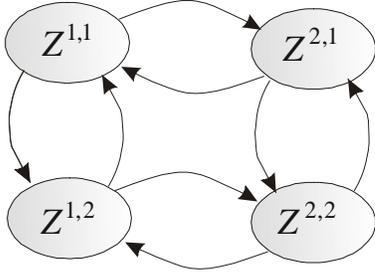


Fig. 3. Graph of functional subnets

4. INVARIANCE OF PROTOCOL

Invariants [3] are a powerful tool of structural properties of Petri nets analysis. It allows the determination of boundness, safeness of net, necessary conditions of liveness and absence of deadlocks. These properties are significant for real-life objects analysis, especially, for communication protocols [1,2].

In the general case *nets with multiply arcs* are considered. It contains an additional mapping $W : F \rightarrow \mathbb{N}$. Multiplicity, in a case it is distinct from unit, is pointed as a number w on the corresponding arc. Let $|P| = m$, $|T| = n$. We enumerate sets of places and transitions. Let us introduce matrices A^- , A^+ for input and output arcs of transitions accordingly:

$$A^- = \left\| a^-_{i,j} \right\|, \quad i = \overline{1, m}, \quad j = \overline{1, n};$$

$$a^-_{i,j} = \begin{cases} w(p_i, t_j), & (p_i, t_j) \in F \\ 0, & \text{otherwise} \end{cases}$$

$$A^+ = \left\| a^+_{i,j} \right\|, \quad i = \overline{1, m}, \quad j = \overline{1, n};$$

$$a^+_{i,j} = \begin{cases} w(t_j, p_i), & (t_j, p_i) \in F \\ 0, & \text{otherwise} \end{cases}$$

And finally, we introduce *incidence matrix* A of Petri net as $A = A^+ - A^-$.

p-invariant of Petri net [3] is a nonnegative integer solution of system

$$\bar{x} \cdot A = 0. \quad (1)$$

t-invariant of Petri net is a nonnegative integer solution of system

$$\bar{y} \cdot A^T = 0.$$

As according to [3] each t-invariant of Petri net is p-invariant of dual net, so further, not limiting a generality, we shall consider only p-invariants.

All known methods of invariants calculation [4,5] have exponential complexity. It makes difficult the application of these methods to real-life objects' models, numbering thousands of elements, analysis.

According to theorem 2 proved in [8], Petri net N is invariant iff all its minimal functional subnets are invariant and moreover exists a common nonzero invariant of contact places. Therefore, to calculate invariants of a Petri net it is required to calculate invariants of its minimal functional subnets and then to find common invariants of contact places. It was shown, that results are true for an arbitrary set of functional subnets defining a partition of the set of transitions of Petri net.

Let a general solution for invariant of functional subnet Z^j has a form

$$\bar{x} = \bar{z}^j \cdot G^j, \quad (2)$$

where \bar{z}^j is an arbitrary vector of nonnegative integer numbers, and G^j is a matrix of basis solutions. Then the system of equations for calculation of common invariants of contact places has a form

$$\left\{ \bar{z}^i \cdot G_p^i - \bar{z}^j \cdot G_p^j = 0, \quad p \in C, \quad (3) \right.$$

where i, j is the numbers of functional subnets, incidental to a place $p \in C$, and G_p^j is a column of matrix G^j , that corresponds to place p .

Therefore, variables \bar{z}^j become not free ones. Note that system (3) has the same form as the source system (1). Thus, it may be solved with above-mentioned methods. Suppose that $\bar{z} = \bar{y} \cdot R$, where R is a matrix of basis solutions of system (3), and \bar{y} is an arbitrary vector of nonnegative integer numbers. So, the general solution of system (1) according to (2) may be represented as

$$\bar{x} = \bar{y} \cdot H, \quad H = R \cdot G. \quad (4)$$

In the cases the model has internal symmetry and owing to what some minimal functional subnets are isomorphic, it is expediently to implement process described above consecutively.

We use the isomorphism of subnets Z^1 and Z^2 . Firstly, we calculate invariants of subnet Z^1 . Then we construct invariants of isomorphic net Z^2 . And finally, we calculate invariant of whole given Petri net.

Invariants of subnets $Z^{1,1}$ and $Z^{2,1}$ we represent as

Result obtained coincides with invariants calculated through usual methods for whole net and also with invariants obtained with direct composition of four minimal functional subnets.

Thus, Petri net is invariant so, for instance, the invariant

$$\bar{x}^* = (2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1),$$

that is the sum of basis invariants with numbers 1, 3 and 9, contains all natural components. Therefore, model of protocol ECMA is safe and bounded.

It should be noted, that though net is also t-invariant one, it contains a deadlock with tokens in places p_9 and p_{11} . Net reaches this deadlock as a result of firing sequence t_1t_5 or t_5t_1 .

5. ESTIMATION OF ACCELERATION

Let us estimate obtained acceleration of computations in the assumption of exponential complexity of algorithms [4,5] for solving of linear Diophantine systems in nonnegative integer numbers. Let the complexity is 2^q , where q is number of nodes of net.

Source net contains 16 places, thus, direct calculation of invariants require solving a system with 16 unknowns. Composition of four minimal subnets requires solving system of the size 7 to obtain invariants of minimal subnets and to solve a system of the size 12 to obtain invariants of contact places. Consecutive composition assumes solving system of the size 7 to obtain invariants of minimal subnets, solving system of the size 5 to obtain invariants of contact places of first composition and solving system of the size 8 to obtain invariants of contact places of second composition. Note that, at the exponential growth of functions, the complexity of matrixes multiplication representing by polynomial of third degree is insignificant and will not be considered.

Complexities of calculation for each of enumerated three ways of invariants obtaining may be estimated by following expressions:

$$S^I = 2^{16} \approx 65000, \quad S^{II} = 2^7 + 2^{12} \approx 4300, \\ S^{III} = 2^7 + 2^5 + 2^8 \approx 500.$$

Thus, decomposition allowed the acceleration more than ten times in the comparison with traditional methods. Moreover, consecutive decomposition allowed the additional tenfold acceleration.

Notice that, accelerations have been obtained for net numbering three tens of nodes. At research of large-scale nets, the acceleration may be rather huge, so it is estimated as exponential function [8].

6. CONCLUSION

Therefore, the decomposition of Petri net model of ECMA communication protocol into functional subnets was implemented. To verify a protocol, Petri net invariants were used. Two ways of protocols' invariants calculation with decomposition into functional subnets were compared: direct and consecutive. Essential acceleration of computations obtained proves the practical value of proposed technique.

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