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CDO rating methodology: Some thoughts on model risk and its implications

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Abstract

Rating collateralised debt obligations (CDOs), which are based on tranched pools of credit risk exposures, does not only require attributing a probability of default to each obligor within the portfolio. It also involves assumptions concerning recovery rates and correlated defaults of pool assets, thus combining credit risk assessments of individual collateral assets with estimates about default correlations and other modelling assumptions. In this paper, we explain one of the most well-known models for rating CDOs, the so-called binomial expansion technique (BET). Comparing this approach with an alternative methodology based on Monte Carlo simulation, we then highlight the potential importance of correlation assumptions for the ratings of senior CDO tranches and explore what differences in methodologies across rating agencies may mean for senior tranche rating outcomes. The remainder of the paper talks about potential implications of certain model assumptions for ratings accuracy, that is the “model risk” taken by investors when acquiring CDO tranches, and whether and under what conditions methodological differences may generate incentives for issuers to strategically select rating agencies to get particular CDO structures rated.

Keywords: Collateralised debt obligations, credit risk modelling, rating agencies.

JEL classification numbers: C15, G11, G15, G20.

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Introduction[1]

Over recent years, collateralised debt obligations (CDOs), ie pools of credit exposure marketed to investors in the form of tranched securities, have become an important part of the global fixed income market.[2] Issuance of these structures has continued to be strong (Graph 1). Traditionally, banks used CDOs to create tranched floating rate securities collateralised by their loans (ie, collateralised loan obligations or CLOs) or other assets that were physically sold to a special purpose vehicle (SPV), which in turn issued several classes of securities against the collateral pool. However, as the market for credit default swaps (CDSs) grew more liquid in the late 1990s, synthetic un- and partially funded CDOs, which acquire credit exposure by writing CDSs, became more popular, particularly in the European market.

From the beginning, the CDO market, much as other markets for securitized products, has been a "rated" market. Issuers of these new products apparently wanted them to be rated according to scales that were comparable to those for bonds, so that investors would feel confident purchasing these structures. As a result, although investors do not appear to exclusively rely on them, ratings are commonly used as benchmarks for assessing CDO investments, with the rating agencies providing a degree of "due diligence" in evaluating these deals. However, as CDOs are based on portfolios of credits rather than a single obligor, rating such structures not only requires attributing a probability of default (PD) to each obligor within the portfolio. It also involves assumptions concerning recovery rates and correlated defaults of pool assets, thus combining credit risk assessments of the individual assets in the collateral pool with estimates about default correlations by way of credit risk modelling.

This degree of complexity, in turn, makes CDO ratings and the methodologies used to assign these ratings an interesting topic for research. In what follows, we will briefly explain one of the most well-known modelling approaches used for rating CDOs, the so-called binomial expansion technique (BET). Using this approach as a benchmark, we will then highlight the importance of correlation assumptions for the ratings of senior CDO tranches.

Based on a comparison of expected loss estimates for a simple CDO portfolio derived by use of the BET and an alternative approach on the basis of Monte Carlo simulation, we then explore what differences in methodologies across rating agencies may mean for senior tranche rating outcomes. The

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2 See Box 1 for a more detailed explanation.

remainder of the paper covers potential implications of certain model assumptions for ratings accuracy, that is the “model risk” taken by investors when acquiring CDO tranches, and explores whether and under what conditions methodological differences may generate incentives for issuers to strategically select rating agencies to get particular CDO structures or tranches rated, ie engage in “ratings shopping”. We close with a short conclusion.

The CDO rating process

The three major rating agencies (ie, Fitch, Moody’s and Standard and Poor’s) subject all products they are asked to rate to a common rating process. Accordingly, rating decisions for traditional bonds as well as structured instruments, such as CDOs, are made by a credit committee on the basis of an assessment of instrument-specific documentation and other analyst-provided information.

However, in the case of CDOs and other structured finance instruments, market participants need to understand not only the default risk embodied in the collateral pool, but also other "non-default" risks arising from the transaction's structure (ie, risks that are not directly related to defaults in the underlying collateral pool, but which affect the credit risk of the tranches). The reliability of a CDO rating, therefore, will depend on the rating agencies' ability to assess the credit risk in the underlying asset pool as well as the accurate modelling of the distribution of cash flows from the asset pool to different groups of note holders. For this purpose, all three major agencies follow a two-stage rating approach:

On the first stage, which is the focus of this paper, analytical models are used to assess pool credit risk. The tools applied for analysing CDO pools may differ according to the nature of the underlying assets and differences will also appear across rating agencies. The second stage of the process is structural analysis. This stage, which will crucially depend on deal specifics as laid out in the CDO’s documentation, involves detailed cash flow modelling as well as legal assessments and evaluations of any third parties involved in the deal, such as servicers and asset managers. The results of the cash flow analysis, in turn, may feed back into the credit model in the form of adjustments made regarding particular model assumptions. Finally, all of the information is aggregated and mapped into a single, alphanumeric tranche rating benchmarked to the historical performance of corporate bonds.[3]

[3] See, eg, Standard and Poor’s (2002) and Bund et al (2003) for detailed descriptions. It is important to note that structural analysis and cash flow modelling, while being abstracted from in this paper, are essential parts of the rating process. Also, all three agencies have repeatedly argued that, depending on the structural features of a transaction, their final rating could be different from what is produced by their models, given that non-quantitative factors might be taken into account. As a result, the analysis in this paper may best apply to synthetic CDOs, which typically do not include structural provisions to the same extent as do funded, cash flow CDOs.

Binomial expansion versus Monte Carlo

The, arguably, most well-known CDO rating methodology is the one based on Moody's primary quantitative approach for generating expected loss (EL) estimates for CDO tranches – the so-called *binomial expansion technique* (BET).[4] The BET was introduced in 1996 and, along with a number of other methodologies, continues to be used in CDO analysis. The method relies on the use of a simple diversification measure, the "*diversity score*" (DS), which is used to map the underlying CDO portfolio into a hypothetical portfolio consisting of DS homogeneous assets. That is, for the purpose of calculating expected loss distributions, the actual portfolio is replaced with a much simpler hypothetical portfolio of homogeneous, uncorrelated securities.

As the number of assets in the hypothetical pool is assumed to equal the diversity score, it will be lower than the number of assets in the actual CDO portfolio to account for the assumption of uncorrelatedness under the BET. Given the homogeneous nature of the hypothetical portfolio, the behaviour of the asset pool can then be described by DS+1 default scenarios (ie, with default occurring for 0 assets, 1 asset, ... DS assets), where the probability of each scenario is calculated using the binomial formula. Having worked out the cash flows (and losses) under each of the default scenarios, these and the default probabilities from the binomial distribution are then turned into estimates of the portfolio and tranche loss distributions.[5]

An alternative methodology, now in use at all three major rating agencies, applies Monte Carlo simulation techniques to estimate the default properties of the underlying CDO asset pool on the basis of repeated trials of random defaults with an assumed correlation structure. For this purpose, default events are simulated within a simplified Merton-type "structural" credit risk model, where default occurs whenever the value of an obligor's assets falls below that of its liabilities. The model's main inputs are asset-level probabilities of default and pair-wise asset correlations, which are turned into an estimate of the entire collateral pool's loss distribution. This distribution is then used, together with other inputs, to determine the required subordination level (ie, level of credit enhancement) for each CDO tranche, given desired tranche ratings. Although MC approaches should produce more accurate loss distribution estimates, they are very computer resource intensive and can take a long time to produce accurate results.

In particular, for cash flow CDOs it is very difficult to build an efficient MC simulation that accounts for all cash flow nuances (eg, different possible "waterfalls"). In some cases, it can take hours for an MC simulation to determine the subordination level for a AAA tranche, and this can be complicated by the tendency for arrangers to change the proposed structures during the rating process. Also, in managed portfolios, the relative value of the

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4 The BET is covered widely in the literature and has thus become one of the main didactical tools for explaining CDO ratings and communicating the principles of structuring CDOs. See, for example, Schönbucher (2003) and Amato and Remolona (2003).

5 See Cifuentes and O'Connor (1996) and Box 2.

simulation approach's asset-by-asset analysis is diminished, while some of the BET's implicit simplifying assumptions (like equal position sizes) closely resemble typical covenants in managed deals.

The choice of the appropriate rating methodology thus involves a trade-off between accuracy and efficiency, the result of which may differ for certain types of CDO structures. This is why Moody's has recently introduced a new Monte Carlo simulation-based methodology, called CDOROM, to rating static synthetic CDOs, while continuing to use the BET and its modifications for rating cash CDOs and managed structures. Some aspects of this decision and its rationale will be explored in this paper.

Moody's Diversity Score: Old versus "alternative" methodology

When applying its BET method, Moody's first calculates the diversity score for the underlying collateral portfolio of the CDO that is to be rated. For this purpose, all credits in the pool are grouped by obligors/issuers and are allocated to the appropriate industry sector (Moody's distinguishes 33 different industry categories). On this basis, the pool's issuer par values and average par (ie, total notional divided by number of issuers) are calculated. The ratios of these two values for each obligor (capped at 1.0 to provide disincentives for issuer concentrations) are then summed up on an industry level. These sums, in turn, are translated into industry diversity scores by use of a table that maps the two scores on the basis of a concave relationship, ie by assuming "diminishing marginal returns" (see Table 1). The sum of all industry diversity scores then gives the diversity score for the total collateral pool.

Mathematically, for a collateral pool of n assets distributed across m industry sectors, the diversity score is hence calculated as follows:

$$DS = \sum_{k=1}^{m} G\left\{\sum_{i=1}^{n_k} \min\{1, F_i / \overline{F}\}\right\},$$

where $\overline{F} = \sum_{i=1}^{n} F_i / N$ (ie, the average holding).

The size of the ith holding is denoted F_i, n_k is the number of assets in the kth sector and G{x} refers to the appropriate entry in Table 1.

In 2000, Moody's started to use an "alternative" diversity score (ADS) instead of the "original" DS described above to rate so-called multi-sector CDOs (ie, CDOs backed by ABS paper). The score is derived by matching the first two moments, ie mean and standard deviation, of the loss distributions associated with the actual collateral pool and the hypothetical, homogeneous portfolio used under the BET. This yields:[6]

[6] See Appendix II of Gluck and Remeza (2000) and Kiff (2004).

$$ADS = \frac{\left(\sum_{i=1}^{n} p_i F_i\right)\left(\sum_{i=1}^{n}(1-p_i)F_i\right)}{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\left(p_i(1-p_i)p_j(1-p_j)\right)^{1/2}F_i F_j}.$$

The formula can be simplified by making assumptions about the uniformity of holding sizes (F_i constant for all i) and default probabilities (p_i the same for all i). Further streamlining can be achieved by assuming that all intra-sector pairwise default correlations are equal to ρ_{int} and all inter-sector pairwise default correlations are ρ_{ext}. With all these simplifications in play:

$$ADS = \frac{n^2}{n+\rho_{ext}n(n-1)+(\rho_{\text{int}}-\rho_{ext})\sum_{k=1}^{m} n_k(n_k-1)}.$$

The new method for calculating diversity scores can now be used to assess the correlation assumptions implicit in Moody's original DS approach. For example, Table 2 shows that a CDO in which all holding sizes and default probabilities are equal would have equal DSs and ADSs for an intra-sector correlation of 20% (and zero inter-sector correlation), if each sector contains 6 holdings, and 30% if each sector contains 3 holdings. Hence, if the pools are homogeneous and evenly spread around, the old DS maps quite closely into the ADS for intra-sector correlations in the 20% to 30% range.

However, if the actual intra-sector correlation is greater than 20%, Moody's old approach runs the risk of underestimating expected losses and over-rating notes by applying a DS that is too high. On the other hand, as conditional probabilities of default are dominated by assumed default correlation, intra-sector correlations in excess of 20% would imply conditional intra-sector PDs of the same order of magnitude. As an assumption like this appears rather conservative, the risk of applying outsized DS estimates may be limited in practice. In addition, Moody's recognizes this shortcoming of its approach and would probably use one of its other methodologies when the BET is deemed not to produce appropriate results.[7]

Comparing the BET and MC approaches

Using the ADS, which explicitly incorporates correlations, it is possible to compare the performance of BET- and MC-based approaches. In addition to generating insights about the impact of correlation assumptions on the credit risk of collateral pools, varying these assumptions then allows for a rough

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[7] For example, for large, very homogeneous pools Moody's will use a "lognormal" methodology, and it uses an MC approach for static synthetic pools and for pools that are very small (<15 assets) or very heterogeneous. For a Moody's methodology "map" see Table 1 in Debuysscher and Szego (2003) as well as Witt (2004).

assessment of Moody's BET methodology vis-à-vis its own CDOROM model and the approaches used by Fitch and S&P.[8]

The simplest version of the BET, as discussed above, defines the expected loss (EL) of the CDO portfolio, on which Moody's will base their tranche ratings, as:

$$EL = \sum_{j=1}^{DS} P_j L_j \quad , \text{ where } \quad P_j = \frac{DS!}{j!(DS-j)!} PD^j (1-PD)^{D-j} .$$

The "diversity score" is denoted DS, PD is the default probability associated with the collateral pool, and L_i denotes the loss in the ith scenario. When Moody's deems the BET to be the appropriate methodology, this simple version is used whenever the collateral pool is fairly homogeneous in terms of size of holdings and default probability distributions. When the pool is made up of two or more uncorrelated groups of assets having "markedly different average properties" Moody's may use a modified version of its standard model, the so-called "multiple BET" (MBET).[9] Finally, a new method, called "correlated binomial", is used for cash flow CDOs backed by pools of highly correlated assets with low diversity scores. Not unlike the ADS, this new approach allows for explicit assumptions about correlation among collateral assets to be incorporated into the underlying credit risk model and will, hence, tend to generate default distributions with higher probabilities of multiple defaults than under the BET.[10]

Table 3 shows the expected losses on a very simple single-period, 10 sector, 6 holdings/sector pool under various assumptions of correlation and subordination level. Overall pool size is assumed to be $600 million, ie 60 individual holdings with the same size of $10 million each and an identical rating of "BB-" on the S&P/Fitch scale.[11] These assumptions, in turn, translate into a diversity score of DS = 30, as in Table 2 (n_k = 6), suggesting that the CDO collateral pool can be meaningfully approximated by a portfolio of 30 homogeneous, uncorrelated securities.

Due to tranching, the subordination level determines the amount of loss protection provided to the senior CDO note holders. Therefore, a protection level of $25 million (4.2% of pool notional) implies that the first $25 million of losses will be borne by the subordinated (ie, equity and mezzanine) note holders, while the remaining $575 million (95.8%) of the pool are being held in the senior tranches. The single-period assumption allows us to ignore various default timing scenarios. Another simplifying assumption is that of zero coupon and discount rates, but this will not detract from the ability to evaluate the

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8 See Box 2 for details.

9 See Cifuentes and Wilcox (1998).

10 See Witt (2004).

11 For a mapping of Moody's ratings into the S&P/Fitch scale, see Jewell and Livingston (1999).

accuracy of the BET approximation relative to the results of a Monte Carlo (MC) simulation. For this purpose, BET-generated ELs for the hypothetical ultra-homogeneous collateral pool described above are compared against those produced on the basis of a simple MC simulation methodology very similar to those used by the major rating agencies.[12]

Importantly, Table 3 documents some of the basic features of CDOs and other tranched credit products. First, the expected loss decreases in the subordination level as any "first losses" are borne by the subordinated tranches, thus providing senior tranche investors with a degree of protection. Second, for any given subordination level, EL increases in default correlation, as probability mass is moved into the tails of the loss distribution. Similarly, EL decreases in pool diversity, as measured by the diversity score. In the extreme, all probability mass is located at the two tails and the portfolio either survives or defaults – essentially resembling the loss distribution of a single asset. As a result, senior tranche investors are said to be "short correlation" in that the value of their tranche decreases as correlation increases (see Graph 2). Third, tranche notionals do not provide meaningful information about credit risk, as the subordinated tranches, though small relative to overall pool size, will take much of the expected loss.[13]

In addition, Table 3 highlights the relative importance of assumed default correlation for the expected losses calculated on the basis of both, the BET and MC methods. In particular, the results suggest that the BET, when compared to MC-based results, accurately calculates EL for pools of homogeneous assets before subordination is considered, and when intra- and inter-sector correlations are zero. On the other hand, it would appear that the BET underestimates EL when the collateral assets are correlated, with the degree of underestimation rising in the subordination level.

This is why, when assigning the rating, Moody's adjusts upwards ("stresses") the portfolio default rate (ie, the portfolio-level default probability based on the collateral pool's weighted average rating (WAR)) to arrive at the default rate entered into the BET formula. The multiplier is 1.5x for Aaa tranches, and slopes down to 1.0x for tranches rated B1 and lower. Hence, whereas a 2.81% WAR was used to generate the senior tranche ELs in Table 3, Moody's would actually employ a probability as high as 4.22% to account for the possibility of larger probabilities of multiple defaults (ie, fatter tails) than assumed under the BET. In fact, although not shown in Table 3, the WAR stresses would, in most cases, result in higher ELs than those produced by the MC-based methodology.[14]

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[12] See Box 3 for details. Moody's idealized default probabilities were used in all the simulations, and a 30% recovery rate was assumed.

[13] See, eg, Gibson (2004).

[14] Use of WAR stresses, however, does not change the fact that the BET is inherently less accurate than the MC method when the pool assets are correlated, given that MC simulation better captures the tails of the loss distribution. This is why, when using the CDOROM model, Moody's does not apply similar stresses.

Table 3 and Graph 3 highlight an apparent link between subordination levels and correlation assumptions on the one hand, and the underestimation of EL on the other. For example, when intra- and inter-sector default correlation are set to 7.56% and 4.29%, the underestimation of EL rises to a factor of 1.26 and 4.0 for subordination levels of $25 and $50 million, respectively. By implication, as the process of tranching distributes EL across CDO notes according to subordination levels, the BET will tend to overestimate EL for the subordinated tranches.

The BET's tendency to underestimate senior tranche ELs arises from the fact that higher default correlations generally translate into lower diversity scores. As the loss distribution can only be specified as multiples of N/DS, where N is the total notional amount, lower DS values will lead to increasingly step-shaped, coarse loss distributions.[15] The effect appears to broadly increase the more a given tranche is located in the right-hand tail of the distribution, thus deflating estimated EL relative to what an MC approach would generate – at least for the senior tranches. In assigning ratings, Moody's compensates for this effect by applying WAR stresses, as described above, and by using a 70% loss-given-default (ie, a "stressed" recovery rate of 30%) in the rating mapping, as opposed to the 55% assumption applied at the individual pool asset level.

Comparing the rating agencies' correlation assumptions

Table 3, when abstracting from differences in recovery rate assumptions, use of stresses and other features of the rating process, also allows for a very basic comparison of the three major rating agencies' approaches towards rating CDOs. In the case of Moody's, assuming a subordination level of $25 million, use of the original diversity score methodology would generate an expected loss of approximately 0.093%, based on a diversity score of DS = 30 and the implicit assumption of inter- and intra-industry default correlation at 0% and 20%, respectively.

S&P and Fitch, on the other hand, who are both using MC methodologies, would likely calculate somewhat different expected losses. S&P's current assumptions regarding asset correlation, given the 2.81% PD implied by the joint BB- rating of the CDO's collateral assets, translate into an implicit 7.56% intra-sector default correlation, while inter-sector correlation is assumed zero. Fitch, in turn, use specific sector vs sector correlations in their MC model and appear to have lowered their assumed average correlations recently. On the basis of their original assumptions, however, implicit intra-sector correlation can be roughly approximated with the same 7.56% assumption employed by S&P, though combined with an implicit 4.29% default correlation on an inter-sector basis. Moody's new MC-based CDOROM model, finally, assumes implicit intra- and inter-sector correlations at 2.96% and 0.48%.[16]

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[15] See Schönbucher (2003), chapter 10.

[16] See Box 3 for an explanation of the link between asset and default correlations. For corporate credits, S&P assume a 30% asset correlation within sector and zero (10% for ABS sectors) between. Fitch is using pairwise asset correlation ranges assigned on the basis of geographical regions and industry sectors, based on observed equity return correlation. As

Given these assumptions, S&P and Fitch would be expected to generate EL estimates at the 0.053% and 0.288% levels, respectively, for a senior tranche with $25 million of subordination. For Moody's, EL levels would fall somewhere in between. While the new CDOROM approach would generate an EL estimate at 0.057%, close to the S&P result, the BET would result in the unstressed EL at 0.093% that was already quoted above. Finally, running MC simulations on the basis of the BET's implicit correlation assumptions, Moody's would likely end up with an estimated EL of 0.118%.

As a result of the diversity of correlation assumptions across rating agencies and CDO methodologies, the ratings that estimated ELs could map into can differ substantially.[17] For example, at the $50 million subordination level, assumed 30% intra- and 0% inter-sector asset correlations (ie, implied default correlations of 7.56% and 0.00%, respectively) result in a 0.000% EL, which would map into a AAA rating. However, if the inter-sector asset correlation is raised to 20% (ie, an implied 4.29% default correlation) the EL rises to 0.068%, which would map into a BBB+ rating.

Model risk and its implications

How important is model risk?

Assuming that investors rely on ratings for their CDO investments, the so-called "model risk" is among the principal risks these investors are exposed to. The risk is related to the specific model the rating agency uses to size the credit enhancement for a given tranche and rating. It will also depend on the agency's correlation and recovery rate assumptions and is, therefore, essentially an issue of rating accuracy. Against this background, it has been argued that the high numbers of downgrades of high-yield CDO tranches over recent years are at least partially the result of under-modelling of both default and recovery rates and, hence, a manifestation of model risk.[18] Investors, therefore, need to understand the model risk they are taking in order to demand appropriate compensation or else risk to earn inappropriate risk-adjusted returns. Put somewhat differently, CDO investors are essentially

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these ranges are difficult to distill down, Fitch's asset correlations were approximated by assuming 30% intra- and 20% inter-sector for the purpose of the analysis in Table 3, although this may be somewhat higher than what is currently assumed by Fitch. Moody's CDOROM model assumes intra- and inter-sector asset correlations at 15% and 3%, respectively. For comparison: The 20% default correlation implied by Moody's traditional DS would map into a 54.75% asset correlation. See Flanagan et al (2004) for a comparative analysis of the EL estimates produced by Moody's old DS-based BET and the new CDOROM methodology.

[17] These results abstract from the fact that S&P and Fitch assign their ratings based on PD, not EL. The analysis also ignores any differences in recovery rate assumptions or other features of the rating process that might affect the ultimate rating. The results should thus be taken as indicative and do not suggest that actual ratings will follow the patterns indicated.

[18] See Adelson (2003), who also notes that default correlation is a time-varying phenomenon. As a result, given that the rating agencies' recovery rate assumptions continue to not fully conform with empirical evidence of substantial cyclical variability in recoveries and negative correlation with default probabilities, their methodologies may not appropriately approximate the tails of pool loss distributions in that systematic risk and, hence, time-variation in correlations and recoveries may not be sufficiently accounted for.

taking an exposure to “complexity” that exposes them to the risk of pool credit risk assessments based on incorrect assumptions. In some sense, higher “complexity” translates into heightened dependence on these assumptions and, thus, higher model risk. As this risk should be expected to be priced by the market, part of the yield pick-up obtained relative to equally rated single obligor instruments is likely to be a direct reflection of model risk.

Table 3 illustrates that correlation effects on estimated EL can potentially be rather large, implying that incorrect assumptions about default correlation can cause the rating agencies to meaningfully under- or overestimate the risk of the collateral pool or any given CDO tranche. For example, for a subordination level of $25 million and assumed asset correlations at 30% intra- and 20% inter-sector (ie, implied default correlations at 7.56% and 4.29%, respectively), the BET-calculated expected loss (abstracting from any stresses that may be applied) will be 0.229%, about 5.5 times higher than the EL for an inter-sector correlation assumption of zero and more than 11 times higher than the EL for a collateral pool with uncorrelated assets. Similar results can be generated on the basis of MC-calculated EL estimates (see Graph 3 for a “measure” of correlation-related model risk on the basis of these considerations).

In the case of Moody’s, given that they are using various versions of the BET and also rely on other methodological approaches, there is also a kind of ”lower-order” model risk, in that the investor depends on Moody’s for the right model choice. Table 3, in turn, implies that this risk can be non-negligible from a tranche perspective, given the variation in EL. The BET result of 0.229% cited above, for example, compares to an EL estimate of 0.288% when using a MC methodology on the basis of identical correlation assumptions. The significance of this effect, as discussed earlier, will tend to rise in the subordination level and for declining pool diversity, with MC-based EL exceeding the BET estimate by a factor of 4 for a subordination level of $50 million and a diversity score of DS = 18.[19]

Ratings shopping: EL versus PD

The possibility of ratings shopping is a related issue, which, similar to model risk-related considerations, also derives from methodological differences in the approaches taken by the major rating agencies in rating CDOs. Amongst credit market participants, it is well known that Moody’s ratings are based on the concept of expected loss, while S&P and Fitch base their ratings on probabilities of default. Accordingly, the relevant result of an agency’s credit risk analysis for a given tranche is ultimately mapped into an alphanumeric scale based on historical (EL or PD) data. As a result, PD and EL ratings provide investors with somewhat different information and should thus be expected to differ for some, if not many, products with multiple ratings. The

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[19] It should be noted that, while strictly a feature only of Moody’s CDO rating approach, a similar “lower-order” model risk may still apply in the case Fitch and S&P. This is because, although both agencies seem to rely on their respective MC-based models for all their CDO ratings, they may still use certain adjustments or customised versions of these standard models for particular types of CDOs or collateral pools with non-standard features.

rating agencies, in turn, have always been careful to communicate to investors both the meaning of their ratings (ie, whether the rating basis is EL or PD) and the methodologies used to assign them.

Nevertheless, methodological differences have led to suggestions that CDO issuers may be incented to “ratings shop”, whereby they “cherry pick” rating agencies based on which one assigns their particular issue or tranche the highest rating.[20] Indeed, Peretyatkin and Perraudin (2002) argue that, given differences in the agencies' rating basis, certain tranche structures can lead to meaningful differences in the ratings assigned by the agencies. In particular, the authors suggest that ratings assigned on the basis of EL tend to be higher than PD-based ratings on “thick” senior tranches and lower on “thin” mezzanine tranches. This, in turn, may lower the funding costs faced by CDO issuers, depending on whether CDO investors understand and see through these possible differences in rating outcomes. As a result, issuers may try to strategically select rating agencies (and adjust the tranche structures of their deals) in an effort to minimize funding costs.

The results in Table 3 are consistent with this finding on the possibility of ratings shopping in that, given different methodologies and correlation assumptions, certain tranche structures can potentially lead to differences in EL estimates. However, the analysis reported in the table focused on the quantitative “engines” with which CDO collateral pools are analyzed by the rating agencies. The results, therefore, are based entirely on EL and reflect differences in modelling, not differences in the rating basis applied by the major agencies.

To analyse the differences implied by EL-based versus PD-based approaches, Table 4 examines the ultra-homogeneous pool presented in the previous section for evidence of any biases; 4a focusing on a high default correlation pool (7.56% intra- and 4.29% inter-sector) and 4b focusing on a zero correlation pool.[21] In each case, two somewhat different two-tranche deal structures are rated on the basis of both, EL and PD. The results suggest that, by choosing appropriate tranche structures, issuers may be able to obtain different ratings, depending on the rating basis applied. Therefore, if an issuer’s objective is to minimize the size of the subordinated tranche, ie to increase deal leverage, the optimal strategy may be to get the senior tranche rated on an EL-basis (ie, by Moody’s).

The “bias” documented in Table 4 is consistent with the results by Peretyatkin and Perraudin (2002), who found that EL ratings were higher on “thick” senior tranches. They also found that the situation was reversed on “thin” mezzanine tranches, for which PD ratings are higher. The results from Table 4 can be extended to show this effect. Table 5 takes the $561/$39 million zero default correlation pool from Table 4b and slices the $39 million

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[20] See, eg, Adelson (1999) and Peretyatkin and Perraudin (2002). Fu (2002) implicitly makes a similar point.

[21] The results in Table 4 are based on the BET methodology and PDs were mapped into ratings using Moody’s “idealized” default rate matrix. In order to map ELs into ratings, the default rates were multiplied by Moody’s standard 55% loss severity rate.

subordinated tranche into ten thinner pieces. The first subordinated tranche, therefore, will absorb the pool's first $25 million of losses, the second subordinated tranche absorbs the next $3 million of losses, and so on down to the tenth subordinated tranche. The senior tranche holds the remainder of the $600 million pool.

On this basis, it turns out that, by "playing around" with the thin tranche sizes, overall deal structure can be adjusted in ways that would make it advantageous for an issuer to seek an EL-based rating for the senior tranche, and a PD-based rating for the rest of the pool. The effect depends crucially on the par value of individual collateral assets, ie $10 million for the pool analysed in Table 5, relative to the size of the deal's mezzanine tranches. Assuming a recovery rate of 45%, the sixth default will push through the 3rd and 4th tranche and eat into the 5th tranche, given that the first five defaults will have wiped out the entire 1st and part of the 2nd tranche. The next, ie seventh, default will then deplete tranches 5 through 9, as the loss given default on the underlying bond, ie $5.5 million, is almost as large as the combined size of these tranches. As a result, the thinner the tranche, the closer EL is likely to be to the PD estimate, given that effective recoveries for some of the mezzanine tranches will be zero. PD-based ratings will thus tend to be favourable for "thin" tranches, providing a potential rationale to "shop" for a PD rating.

Real-world "biases" in CDO rating patterns?

The real-world importance of ratings shopping, as pointed out earlier, will crucially depend on at least two factors. First, the extent to which differences in agencies' recovery rate assumptions or in structural analysis work to correct the effects shown above. Second, the degree to which CDO investors are or are not able to see through such a strategy on the part of issuers. In that sense, documented "biases" in agencies' analytical approaches can be seen as a necessary, but not sufficient, condition for ratings shopping to be a possible strategy for issuers.

In fact, the limited empirical evidence available seems to suggest that ratings shopping is likely not to be a significant phenomenon in practice. CDOs (at least outside the market segments serving the most sophisticated CDO investors) are commonly regarded as a "two ratings market". Significant differences in credit opinion across rating agencies would thus tend to be filtered out in the case of multi-rated tranches.[22] Indeed, a recent study of disclosed ratings for US$-denominated structured finance deals, NERA (2003), finds that, in recent years, around 90% of the multi-rated CDO tranches in their sample involved a Moody's rating along with ratings by one or two of the other major rating agencies. Investors, to the extent that they relied on multiple ratings, thus appear to have preferred both a PD and EL-based rating (see Table 6). Ratings differences on these jointly rated deals were found to be rather small, with average differences at issuance at 0.15 notches and below, depending on the rating agency pairs involved.

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[22] See Fu (2002).

While this argues against substantial "biases" in CDO ratings, low ratings are likely to be foregone by the issuer, meaning that observed rating differences will be deflated relative to what might have been realised on the basis of ratings assigned by randomly chosen agency pairs. Consistent with this and the methodological differences documented above, NERA appears to find Fitch to be somewhat more likely than the other rating agencies to solely rate the most subordinated tranche in jointly rated transactions. The other agencies, in turn, appear to be more likely than Fitch to rate the most senior tranche, while Moody's was somewhat more likely to solely rate the highest senior class than both Fitch and S&P.[23]

Against this background, if issues were expected to arise, these would likely be linked to situations where single-rated deals became more common (as, for example, in the synthetic market) – at least to the extent that the respective investors were unable to perform their own risk analysis. According to the NERA sample, the number of single-rated tranches has increased substantially between 1995 and 2001. At the same time, however, the share of multi-rated tranches has steadily increased from less than 5% in 1995/96 to nearly 30% by end-2001. Overall, this may suggest that any biases emerging from differences in rating agencies' approaches have become less of an issue over recent years.

Conclusion

This article documents some of the key features of the rating agencies' models for evaluating CDO collateral pool credit risk and how differences in model specifics may influence the credit risk assessment of individual pool tranches. It is shown that use of different modelling approaches may, in theory, lead to different rating outcomes for individual tranches, particularly once differences in correlation assumptions are taken into account. This may have important implications for CDO investors and originators. At the same time, however, it should be noted that the simulations in this paper are based on simplified examples, ie abstracting from the specifics of cash flow analysis, any structural enhancements, differences in recovery rate assumptions and the like. Results, therefore, need to be interpreted with caution and can not be seen as proof of any real-world rating patterns across rating agencies.

Nevertheless, a number of interesting insights emerge from the analysis. First, the results highlight the importance of correlation assumptions for expected loss estimates and, potentially, CDO tranche ratings. Getting these assumptions right, therefore, is one of the key challenges for the rating agencies in dealing with pooled credit risk and decisive for ratings accuracy. Differences in correlation assumptions and modelling approaches can, when combined, potentially lead to meaningful differences in tranche ratings, unless compensated by any differences in other parts of the rating process, such as recovery rate assumptions or adjustments made in response to specific

[23] NERA (2003) note, although detailed results are only reported for the entire structured finance universe, that this ratings pattern holds across all products in their sample.

structural features of the rated instrument. The resulting “model risk” needs to be understood by investors and argues against exclusive reliance on CDO ratings in taking investment decisions. In addition, continuing investor demand for more than one rating per tranche may be justified to help avoiding inappropriate risk-adjusted returns.

Second, to the extent that investors do not fully understand the possible implications of these effects for tranche ratings, ratings shopping is a theoretical possibility. That is, originators may be tempted to minimize their funding costs by tailoring deal structure and strategically selecting rating agencies to obtain favourable ratings on particular tranches. Incentives for such a behaviour may arise from differences in modelling pool credit risk, from differences in the rating basis applied across rating agencies or from combinations thereof. Evidence of this sort of strategy being applied in practice, however, is limited, suggesting that the methodological differences shown above are at least partially ironed out elsewhere in the rating process or that investors “see through” the incentives that may arise in this context. Nevertheless, while the scope for ratings shopping should not be overstated, insistence on multi-rated tranches, combined with investor due diligence, may help to avoid disappointment.

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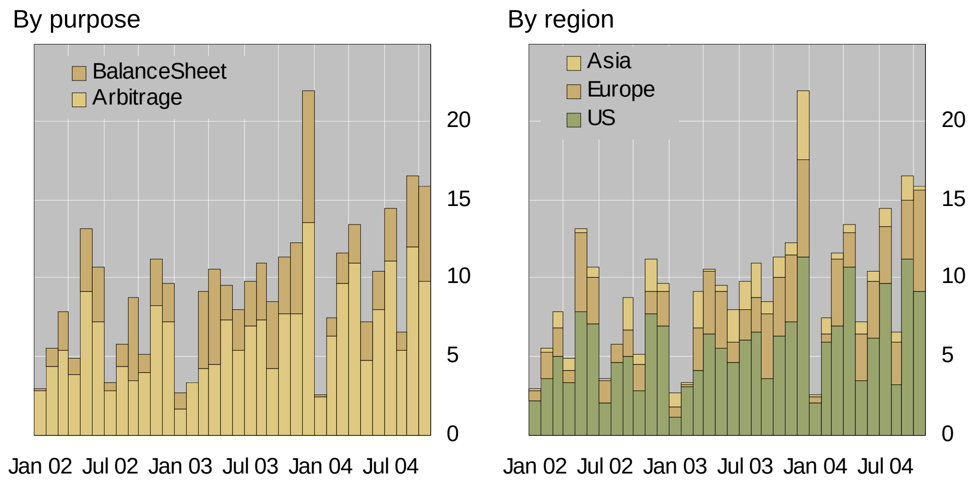
Exhibits

Global CDO funded issuance

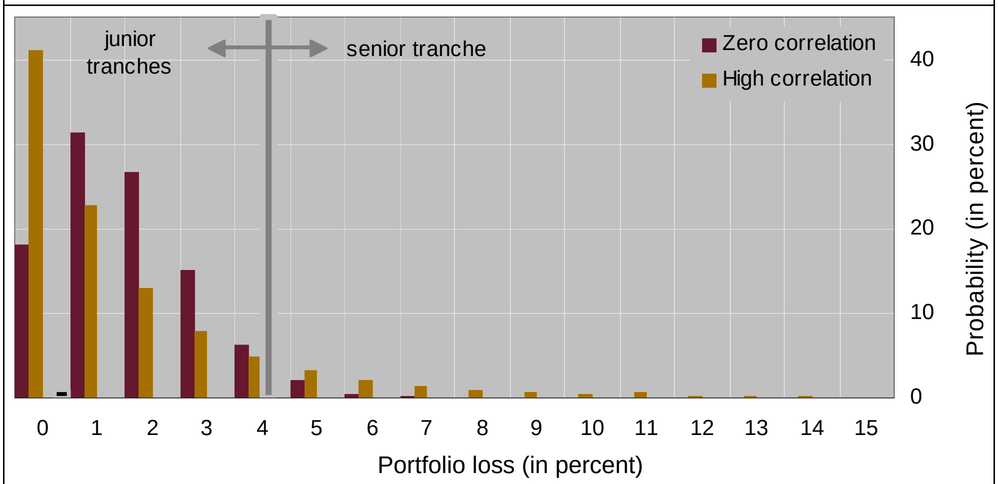
In billions of US dollars; includes cash issuance and funded portion of synthetics



Source: JPMorgan Chase. Graph 1

Portfolio loss distributions: Zero versus high correlation

Losses on ultra-homogeneous US$ 600 million pool[1]



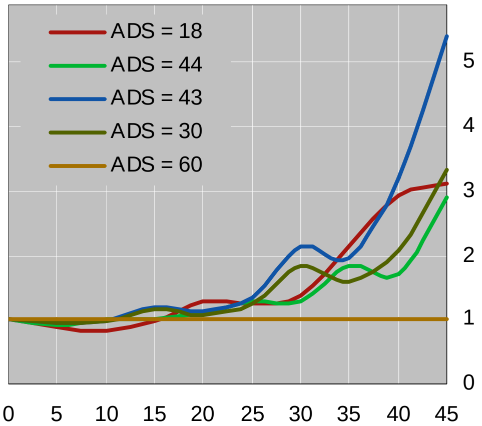
[1] EL = 1.546%; high correlation pool assumes 30% intra- and 20% inter-sector asset correlation.

Source: Authors' calculations. Graph 2

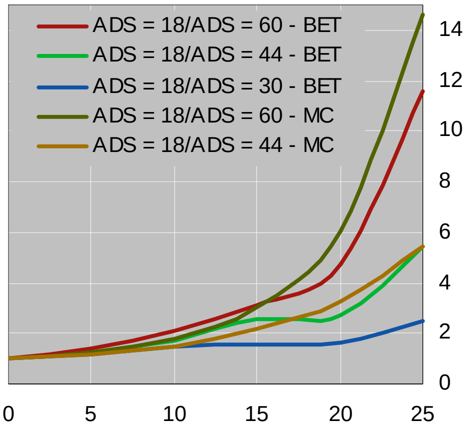
EL estimates by methodology, pool diversity and subordination

Horizontal axis: Subordination levels in US$ millions

MC/BET-based EL ratios[1]



Pool diversity-based EL ratios[2]



[1] EL on MC- and BET-basis for subordination levels between $0 and $45 million. [2] EL based on ADS=18 relative to score given for different methodologies.

Source: Authors' calculations. Graph 3

Moody's diversity score table

Unit score (x)	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	>6.0
Diversity score (G{x})	1.0	1.2	1.5	1.8	2.0	2.2	2.3	2.5	2.7	2.8	3.0	TBD

Source: Moody's. Table 1

ADS versus DS under various intra-sector default correlations

	Number of holdings per sector		Intra-sector default correlation (Inter-sector correlation = zero)						
			0%	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**
Sectors(m):10 Holding(F_{avg}):10	n_k=6	ADS	60	48	40	34	30	27	24
		DS	30						
	n_k=3	ADS	30	27	25	23	21	20	19
		DS	20						

Source: Moody's; authors' calculations. Table 2

BET vs MC: An ultra-homogeneous $600 million asset pool

Sector Default (Asset)[1] Correlation		Diversity Score	Subordination Level	Expected Loss on Senior Tranche		
Intra	Inter			BET	MC	MC/BET
0% (0%)	0% (0%)	60	$0	1.546%	1.546%	1.00
			$25	0.020%	0.020%	1.00
			$50	0.000%	0.000%	-
7.56% (30%)	0% (0%)	44	$0	1.546%	1.546%	1.00
			$25	0.042%	0.053%	1.26
			$50	0.000%	0.000%	2.85
2.96% (15%)	0.48% (3%)	43	$0	1.546%	1.546%	1.00
			$25	0.042%	0.057%	1.36
			$50	0.000%	0.001%	8.80
20% (55%)	0% (0%)	30	$0	1.546%	1.546%	1.00
			$25	0.093%	0.118%	1.28
			$50	0.002%	0.004%	2.51
7.56% (30%)	4.29% (20%)	18	$0	1.546%	1.546%	1.00
			$25	0.229%	0.288%	1.26
			$50	0.017%	0.068%	4.00

[1] The corresponding asset correlations (in parentheses) are calculated at the pool's homogeneous 2.81% default probability.

Source: Authors' calculations. Table 3

EL- versus PD-based ratings on ultra-homogeneous asset pool

Correlations: Intra-sector = 7.56%, Inter-sector = 4.29%. Tranche size in US$ millions.

Senior Tranche				Subordinated Tranche			
Size	**Basis**	**EL/PD estimate**	**Rating**	**Size**	**Basis**	**EL/PD estimate**	**Rating**
$522	EL	0.000314%	AAA	$78	EL	11.885992%	CCC+
	PD	0.011046%	A		PD	40.132865%	CCC
$490	EL	0.000001%	AAA	$110	EL	8.429735%	B-
	PD	0.000033%	AAA		PD	40.132865%	CCC

Source: Authors' calculations. Table 4a

EL- versus PD-based ratings on ultra-homogeneous asset pool

Correlations: Intra-sector = 0%, Inter-sector = 0%. Tranche size in US$ millions.

Senior Tranche				Subordinated Tranche			
Size	**Basis**	**EL/PD estimate**	**Rating**	**Size**	**Basis**	**EL/PD estimate**	**Rating**
$561	EL	0.000291%	AAA	$39	EL	23.771242%	CCC
	PD	0.027007%	A		PD	81.915982%	CCC-
$545	EL	0.000001%	AAA	$55	EL	16.858932%	CCC
	PD	0.000083%	AAA		PD	81.915982%	CCC-

Source: Authors' calculations. Table 4b

Ratings shopping with zero default correlation

Correlations: Intra-sector = 0%, Inter-sector = 0%. Tranche size in US$ millions.

Tranche	Size	EL-basis		PD-basis	
		Expected Loss	Rating	Default Probability	Rating
1st	$25	36.637877%	CCC-	81.915982%	CCC-
2nd	$3	2.336643	BB-	2.669211	**BB**
3rd	$2	0.673804	BB+	0.673804	**BBB-**
4th	$2	0.673804	BB+	0.673804	**BBB-**
5th	$2	0.409382	BBB-	0.673804	BBB-
6th	$1	0.144960	BBB	0.144960	**BBB+**
7th	$1	0.144960	BBB	0.144960	**BBB+**
8th	$1	0.144960	BBB	0.144960	**BBB+**
9th	$1	0.144960	BBB	0.144960	**BBB+**
10th	$1	0.085983	BBB+	0.144960	BBB+
Senior	$561	0.000291	**AAA**	0.027007	A

Source: Authors' calculations. Table 5

Number and percentage of CDO tranche ratings by rating agency

Disclosed ratings of tranches of US$-denominated deals for the period 1995-2001

Year end	Tranches rated by						
	Fitch only	Moody's only	S&P only	Moody's and S&P	Fitch and Moody's	Fitch and S&P	All three agencies
1995	0 (0.0%)	164 (87.2%)	22 (11.7%)	2 (1.1%)	0 (0.0%)	0 (0.0%)	0 (0.0%)
1996	2 (0.9%)	171 (81.0%)	31 (14.7%)	5 (2.4%)	2 (0.9%)	0 (0.0%)	0 (0.0%)
1997	21 (4.8%)	294 (67.4%)	65 (14.9%)	19 (4.4%)	21 (4.8%)	9 (2.1%)	7 (1.6%)
1998	66 (8.7%)	451 (59.1%)	116 (15.2%)	52 (6.8%)	47 (6.2%)	24 (3.1%)	7 (0.9%)
1999	105 (8.3%)	657 (52.1%)	185 (14.7%)	168 (13.3%)	90 (7.1%)	35 (2.8%)	21 (1.7%)
2000	159 (9.0%)	883 (50.0%)	230 (13.0%)	236 (13.4%)	164 (9.3%)	53 (3.0%)	42 (2.4%)
2001	228 (9.7%)	1200 (51.2%)	258 (11.0%)	275 (11.7%)	282 (12.0%)	52 (2.2%)	48 (2.0%)

Source: NERA (2003). Table 6

Box 1: Collateralised debt obligations (CDOs)

A CDO is a structured finance product in which a distinct legal entity, a *special purpose vehicle* (SPV), issues bonds against an investment in an underlying asset pool. Pools may differ with regard to the nature of their underlying assets and can be collateralised either by a portfolio of bonds, loans and other securities, or be backed by synthetic credit exposures, ie via use of credit derivatives and credit-linked notes. (More recent structures, such as single-tranche CDOs, may no longer rely on SPVs, but otherwise use the same structuring technology).

The claims issued against the collateral pool of assets are prioritised in order of seniority by creating different tranches of debt securities, including one or more investment grade classes and an equity/first loss tranche. Senior claims are largely insulated from default risk to the extent that the more junior tranches absorb credit losses. As a result, each tranche has a different priority of payment of interest and/or principal and may thus have a different rating.

See CGFS (2003) for more detail on CDOs and their economics.

Box 2: CDO rating methodologies – a broad overview

CDO rating methodologies used by the three major rating agencies Fitch, Moody's and Standard & Poor's are broadly similar, but important differences remain. Moody's established BET methodology is top-down, ie portfolio based, while Fitch and S&P now use methodologies more geared towards the asset level. All three agencies have revamped their methodologies in the recent past or are in the process of doing so, which has led to substantial changes in the way pool credit risk is evaluated and ratings are assigned. However, all of their methodologies essentially attempt to capture the credit risk of CDOs by making estimates of or assumptions about individual default and recovery rates and about pairwise default correlations among obligors in the portfolio. Furthermore, assumptions about default times and similar deal features are important inputs into the approaches applied by the rating agencies.

Moody's continues to use its widely known BET model, although new Monte Carlo simulation-based methodologies have recently been introduced for static synthetic CDOs and CDOs-squared. For the purpose of calculating expected loss distributions, the BET model maps the CDO collateral pool into a hypothetical portfolio consisting of DS uncorrelated, homogeneous assets with identical default probabilities (assumed to equal the weighted average probability of default of the original pool) and equal par values. The number DS of securities in this hypothetical portfolio is assumed to be equal to the so-called "diversity score", which is a simple measure of diversification. Moody's method for calculating DSs, the main input into the BET model, has now been refined to explicitly account for default correlation. In addition, Moody's is also using other methods, such as the MBET and the Correlated Binomial. Its new CDOROM Monte Carlo model is similar to what's applied by S&P and Fitch in that explicit inter- and inter-sector correlation assumptions are being fed into a simulation engine. In addition, the model also simulates correlated recoveries to account for systematic variation.

Fitch has recently revamped its CDO rating methodology by introducing its VECTOR model, which estimates CDO portfolio default distributions on the basis of Monte Carlo simulations. Default rates and asset correlations are inputs into the model. The default rates come from a new CDO Default Matrix (giving asset default rates by rating and maturity), which is based on historical bond default rates and can be modified to take account of "softer" default definitions when used for rating synthetic CDOs. Pairwise asset correlations, similar to what's done by Moody's, are based on estimates of cross- and intra-industry, and geographical correlations of equity returns. As a result, Fitch will assign an internal and external correlation for each of the 25 industry sectors used. In the past, Fitch did not explicitly model correlations, but applied penalties for high obligor, industry and country concentrations in CDO collateral pools.

Standard and Poor's introduced its EVALUATOR model back in 2001. The model is based on Monte Carlo simulations, taking the PD and rating for any name in the pool and correlations between pairs of assets into account. The simulation engine draws large numbers of multivariate normally distributed numbers, which are then compared with a default threshold (based on the maturity and PD for the asset) to decide whether a given asset defaults or not. Being MC-based, the model is broadly similar to VECTOR, but with subtle differences. Both models follow a two-step process, ie various probabilistic inputs are being calculated in one system and then fed into a separate cash flow model – the same applies also to Moody's approach. However, while S&P's EVALUATOR is based on a one-period simulation, Fitch's VECTOR model computes the default distribution by use of a multi-period simulation. S&P's correlation assumptions are based on historically observed defaults, with asset correlation then calibrated to default correlation observed over the cycle, while Moody's and Fitch use assumptions based on equity returns as inputs for their respective MC models.

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See, eg, Cifuentes and O'Connor (1996), Bund et al (2003), and Standard and Poor's (2003) for more detail.

Box 3: The Monte Carlo approach and asset versus default correlation

The Monte Carlo (MC) methodologies used by the rating agencies are implicitly based on the simulation of default events within a simplified "structural" credit risk model. Basically, they assume that default occurs when the value of an obligor's assets falls below that of its liabilities. All assume that changes in obligor asset values are lognormally distributed so that a normalized "distance" to the default "threshold" (DD_i) can be inferred from the default probability (PD_i) associated with the obligation's credit rating: $DD_i = N^{-1}(PD_i)$, where $N^{-1}(x)$ is the inverse of the standard normal distribution.

In each simulation run, a correlated standard normal random variable is drawn for each obligation in the portfolio, which is taken to represent the normalized change in the obligor's asset value over the appropriate horizon (ΔX_i). If $\Delta X_i < -DD_i$, a default is indicated and a loss or recovery is drawn from an appropriate distribution. Default losses are then accumulated for the n assets in the pool to arrive at a total loss (L_T):

$$L_T = \sum_{i=1}^{n} \left(\Delta X_i < -DD_i\right) V_i \left(1 - RR_i\right)$$

where RR_i is the recovery rate on the ith obligation and V_i is the value of the ith obligation. The loss on the kth tranche (L_k) is then defined by the following function:

$$L_{k,j} = \min\left\{\max\left\{L_j - \alpha_{k-1}, 0\right\}, \alpha_k - \alpha_{k-1}\right\}$$

where α_{k-1} is the kth tranche's subordination level (or "attachment point") and α_k is the "detachment point" beyond which losses are absorbed by the more senior tranches. In other words, the attachment point is the total outstanding value of all the more junior tranches, with ($\alpha_k - \alpha_{k-1}$) defining tranche "thickness" - the loss absorption capacity of the tranche.

For simulating the correlated standard normal variables, it is important that the correlation of the underlying assets be used, as opposed to the default correlations applied in Moody's ADS-based BET methodology. The correlation between two discrete default events is defined as:

$$\rho_{ij} = \frac{PD_{ij} - PD_i PD_j}{\sqrt{PD_i\left(1 - PD_i\right)PD_j\left(1 - PD_j\right)}}$$

where *PD_{ij}* is the joint probability of the default of obligations *i* and *j*. In the context of the structural default model that is being applied to individual obligations, *PD_{ij}* is has to be equal to the probability of the value of the two assets both declining by more than their respective distances to default:

$$PD_{ij} = N_2^{-1}\left(\Delta X_i < -DD_i, \Delta X_j < -DD_j, \rho_{ij}^V\right)$$

where $N_2^{-1} = (\ )$ denotes the inverse cumulative bivariate normal distribution and ρ_{ij}^V is the asset correlation. On this basis, default correlations will be lower than asset correlations, but will increase with the respective assets' individual default probabilities up to the 50% PD level and then decline symmetrically. As a result, a 30% asset correlation, assuming identical PDs for any pair of assets, will translate into default correlations in a 4.6% to 19.4% range, depending on PD levels. A 20% asset correlation, in turn, corresponds to default correlations between 2.41% and 12.8%.

See Morokoff (2003) and Kiff (2004) for details.

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