Chapter 1
History of Geodesy

Stonehenge in UK

Egyptian Pyramids

Ever since man evolved into a thinking creature, he has been interested in learning about the earth. The various natural phenomena he observed around him, often with awe of fear, were responsible for his behaviour and gave rise to various superstitions. These, in turn, encouraged a better comprehension of events which resulted in many early cultures and civilizations acquiring a suprisingly deep understanding of some of the natural phenomena, left to us in such obvious forms of as monuments (like Stonehenge, Egyption Pyramids). The natural phonomena are related to the size, shape, gravity field of the earth. To understand them it needs GEODESY.

Together with astronomy, geodesy is among the the oldest sciences, it is the oldest geoscience.

The development in geodesy can be divided into four chronological sections:

1- Period from Thales till the end of Roman Empire
2- The Middle Ages, The Renaissance till the mid-eighteenth century.
3- Next 200 years, ending with second World War
4- The most recent developments.

1.1 Historical beginnings of Geodesy:

During the Greek era, geodesy was considered to be one of the most challenging disciplines. Thales of Miletus (625-447 B.C.) involves the first documented ideas about Geodesy. There are several ideas about the earth's shape. (Figures1.1-1.3)

Around the end of the 6th century the first known maps of the world was compiled. The first star maps was prepared by Eudoxus (408-355 B.C) who also knew the length of the solar year almost exactly 365.25 days. (Figure1.4)

Aristoteles (384-322 B.C) formulated the argument for the sphericity of the earth and first hints of gravity was considered.

Around the end of the 3rd century the spherical coordinates were introduced. Aristarchus (310-250 B.C.) attempted to determine the dimensions and distances of the moon and the sun. About half a century later, the motion of the obliquity of the earth's spin axis was introduced.
Eratosthenes can be called the proper founder of geodesy. (Figure 1.5)

1.2 Scientific beginings of Geodesy

In the centuries following the fall of Roman Empire, during the Middle Ages, geodesy came more and more within the detrimental embrace of theology. (Figure 1.7)

The major explorations got under way at the end of the fifteenth century with Columbus (1492), Vasco da Gama (1497), Magellan (1519). The expanding geographical knowledge prompted the growth of a new profession: map making and cartography. Cartography is the art of displaying the final product of geodesy. Amerigo Vespucci (1451-1512) prepared the first maps of North America Pacific coast and provided a name for the continent. Mercator can be considered to be the father of modern cartography.

Indications of an impending revival of geodesy can be found in the mid-fifteenth century. (Copernicus, Kepler,).
Galileo (1564-1642) progress in theory
Kepler(1571-1630) and the invitation of telescope.

Gravity was introduced by Stevin (1548-1620).
Snell (1591-1626) carried out the first accurate triangulation and study of refraction.
Picard in 1670 made the first modern measurement of the earth (6275 km).
The Newton's law of the universal gavitational attraction was the most important discovery in this era (1687).

1.3 Geodesy in the service of mapping

Networks of points whose horizontal positions were determined from the measurements of angles and occasional distances, known as triangulation networks, was started to be used to support the mapping.

Laplace: foundations for modern celestial mechanics, theory of tides.
Gauss: defined the geoid, invented the least square methods.

In the 19th century, most of the tools applied mathematics used in geodesy today were invented. Euler (1707-83), Lagrange (1736-1813), Fourier(1768-1830).

The beginning of the 20th century Eintein's theory of relativity (a further generalization of Newton's theory of gravitation) affects the thinking of physicits.

1.4 Geodesy of Modern era

The mid 20th century saw the dawing of the technological revolution. Prompted by weapons and defence requirements during the 2nd World War, the invention of a radio detection and ranging system, radar, has had a deep effect on the philosophy behind geodetic instruments. Developments in the computers provide numerical calculations. After the war accurate electromagnetic measuring devices became commercially available for geodetic uses. They first used polarized light, then radiowaves and finally lasers.

The launching of the first satellites was another giant leap for geodesy. Satellites also brought about a new project for geodesy: The mapping of the gravity field above the earth to predict the satellite orbits.

The increased ease and accuracy with which geodesists could determine positions, as well as, the gravity field parameters, led to new applications, but also to new problems.

The last important development of geodesy concerns the sea. It helps to satisfy the steadily growing demand for accurate navigation.

2. Geodesy and other disciplines

It is beneficial to know the relationships of geodesy to other disciplines.

2.1. Applications of geodesy

Surveying is the practice of positioning , and geodesy is the theoretical foundation of surveying. For centuries, the role of geodesy was to serve mainly mapping.

a) Mapping: There is a need for an areal network of appropriately distributed points (geodetic control) of known horizontal and vertical positions for the production of maps.

b) Urban Management: The locations of man's creations must be defined and documented for future reference.
c) Engineering projects: Coordinates of large structures, like dams, bridges must be obtained according to control points. In the case of dam, irrigation channels, the exact shape of equipotential surfaces of the gravity field should be known.
d) Boundary demarcation: Oil and gas leases in remote and inhospitable parts of the word can be positioned by relating them to a framework of point with known horizontal coordinates.
e) Ecology: Movement of ground caused by the removal of underground resources or subsurface disposal of wastes. The detection and monitoring of these movements is a geodetic problem.
f) Environmental Management: Transportation, land use, community and assesment of tax data and population statistics should be based on land parcels whose locations are uniquely defined in terms of coordinates.
g) Geography : All the positional information needed in geography is provided be geodesy.
h) Planetology: It uses methods for studying the geometry, gravity fields and deformations of planets. Practically all of geodesy is applicable to planetology.
i) Hydrography: Positioning at sea, combined with the depth sounding, and applies many geodetic methods.

2.2. Symbiotic relation between geodesy and some other sciences:

There are many more uses for geodesy then simply mapping. There are some other scientific fields thet have a two-way relation with geodesy.

a) Geophysics: It deals with the physical response of the earth to a variety of forces, the internal structure of the earth affecting its motion. This information is needed when various methematical models for geodetic purposes are being designed. Gravity is one of the most important sources of information used in geophysics. It also needs positions and other geometrical information geodesy can supply.
b) Space Science: It needs the knowledge of the geometry of the earth's external gravity field for predicting the orbits of the space vehicles. Space science has developed some very powerful positioning systems that use the earth's artificial satellites, and those are being used in geodesy.
c) Astronomy: Of common interest is the monitoring of the rotation of the earth. An other part of astronomy , celestial mechanics is also needed in geodesy to study the satellite orbits.
d) Oceanography: Both are involved in the location and movements of shorelines. Geodesy provides relative heights of the on-shore water level measuring devices and their relative vertical movement. Oceanography provides the deviations of the mean sea surface from an equipotential surface of the earth's gravity field. This information is needed for the establishment of a datum for heights.
e) Atmospheric Science: It looks the effect of the distribution of air density. Geodesy needs realistic models for atmospheric refraction which represents one of the most troublesome problems in many geodetic measurements.

f) Geology: requires both horizontal and vertical positions for its maps. It provides geodesists with a knowledge of geomorphology and local stability of different geological formations.

2.3. Theoretical basis of geology:

These disciplines provide the theoretical basis for geodesy.

a) Mathematics: The most important building block of geodesy. Algebra, Analysis, Geometry, Statistics are used.
b) Computer Science: Many of the problems faced by geodesy require a computer solution. Numerical analysis concepts are needed in geodesy.
c) Physiscs: Gravitational has played a very important role in geodesy. Gravity is the geometry of the space in which most geodetic observations are taken. Mechanics is required to understand the motions of the earth and its satellites.

2.4. Functions of Geodesy

Geodesy was defined as 'Geodesy is the science of measuring and portraying the earth's surface' until a decade or two ago.

Geodesy is the discipline that deals with the measurement and representation of the earth, including its gravity field, in a three dimensional time varying space'.

- Geometrical geodesy
- Mathematical geodesy
- Physical geodesy
- Dynamic geodesy

<u>Functions of geodesy:</u>

a) Positioning
It is a geodetic task. Points can be positioned either individually or as a part of a whole network of points.

b) The earth's gravity field
The knowledge of the earth's gravity field is needed to make possible the transformation of the geodetic observations made in the physical space , affected by gravity, into the geometrical space in which positions are usually defined.

c) Temporal variations
Temporal variations of positions and the gravity field result from deformations of the earth attributed to a number of causes. The study of these causes belongs to the geophysics.

The goals of GEODESY:

1- Establishment and maintenance of national and global three-dimensional geodetic control networks on land, recognizing the time-varient aspects of these networks.
2- Measurement and representation of geodynamic phenomena (polar motion, earth tides)
3- Determination of the gravity field of the earth including temporal variations.

Chapter 2
Earth and Its Gravity Field

Instruments with which geodetic measurements are made, on and above the surface of the earth, are subjected to various physical forces. To interpret the results of the measurements properly, it is necessary to understand the effects of these forces.

Gravity is the most conspicious force present on the surface of the earth. Since basic geodesy deals with either stationary or slow moving objects, the gravitational theory needed is that of Newton rather than Eintein. The gravity field on and immediately above the earth surface is dealt with the earth is regarded as a rigid body.

In the first section, the earth's gravity field is defined from the physical and mathematical points of view. The next section is devoted to a description of the magnitude of gravity and how it is handled. In the third section, gravity potential is explained and the terms of equipotential surface and plumb lines are defined.

2.1 Gravity Field

Newton's (1687) law of universal gravitational, states that a body of mass M attracts another body of mass m by a force F proportional to the product of the two masses and inversely proportional to the square of their distance Δr :

$$F = G\frac{Mm}{\Delta r^2} \tag{1}$$

This force is known as the gravitational force (gravitational attraction). The constant of proportionality G is known as gravitational constant.

$G = 6.67 * 10^{-8}$ $g^{-1}cm^3s^{-2}$ (experimentally determined)

Taking the bodies A, B with masses m, and M and considering their dimensions negligibly small compared with their distance, the following vector equation can be written (Fig 2-1) for the gravitational force that B exerts on A.

$$\vec{F}_{B\to A} = G\frac{Mm}{\left|\vec{r}_B - \vec{r}_A\right|^3}(\vec{r}_B - \vec{r}_A) \qquad \vec{e}_1 = \frac{\vec{l}}{\left|\vec{l}\right|} \tag{2}$$

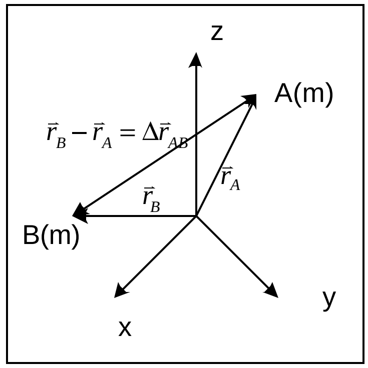


Figure 2.1 The gravitational attraction btw two bodies.

If the dimensions of one of the two bodies can not be regarded as negligible: small body A, the earth B.

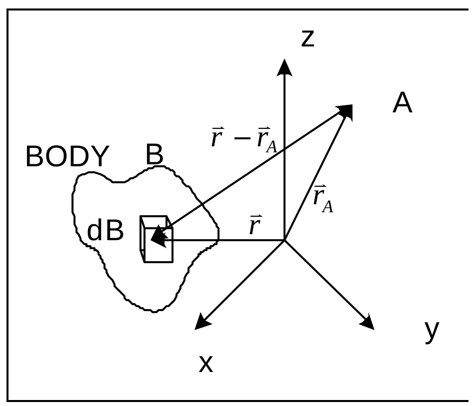


Figure 2.2. Gravitational attraction of a physical body.

Body B is thought as being composed of a number of small massive elements of volume dB, and the attraction of each of these on A can be investigated separately. (Figure 2.2)

The mass density within the body σ(r) and dB is chosen small enough so that σ and dB can be considered constant.

$$\vec{F}_{dB\to A} = G\frac{s(\vec{r})(dB)m}{\left|\vec{r} - \vec{r}_A\right|^3}(\vec{r} - \vec{r}_A) \tag{2.3}$$

Gravitational forces are additive (experimentally determined). This means that the sum of forces produced by the elements dB is equal to the force exerted by the whole body B.

$$F_{\vec{B}\to\vec{A}} = \vec{F}(\vec{r}_A) = Gm\iiint_B \frac{s(\vec{r})}{\left|\vec{r} - \vec{r}_A\right|^3}(\vec{r} - \vec{r}_A)dB \tag{2.4}$$

To study the gravitation, the density distribution σ(r) within the earth must be known. But such a distribution is known only approximately. From seismic observations one of the existing density distribution models is given in Figure 2.3. All of the seismic models assume a perfectly spherical distribution, so the density is a function of depth only. (distance from the center of mass). The gravitational force produced by such a model earth is radial. The force generated by this body always points towards the centre of mass.

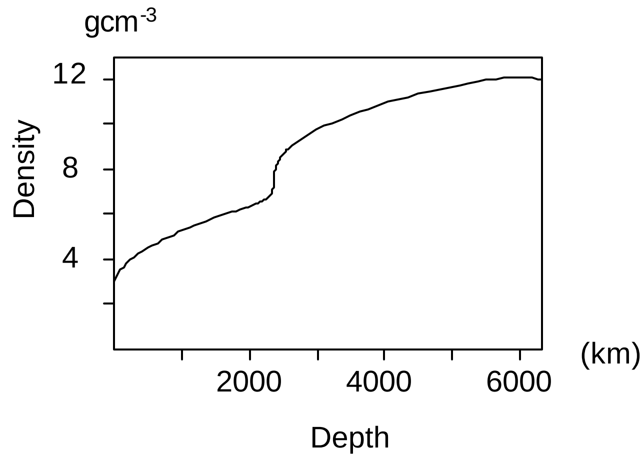


Figure 2.3 Variation of density with depth.

If GM=3.98603*10^{20} cm^3/s^2, R=6371.031 km then eqn 2 gives the mean value of the gravitational attraction on the surface of the earth.

$|\vec{F}| = F = 982.022$ (cm/s^2)*m where m = mass of the attracted body (2.5)

Since the real distribution of density within the earth is not only radially but also laterally irregular and the earth is not spherical, the gravitational force field is not perfectly radial either. The value given by 5 is only a mean global value. When the density varies with time, so does the gravitational force. Real variations are minute and difficult to detect. In all geodetic work the practice has been to ignore these variations with the exception of the tidal variation.

Even if the earth is assumed to be rigid, it is spinning. The spin of the earth causes an additional force, centrifugal force. Its direction is always perpendicular to the instantaneous spin axis.

The magnitude of the centrifugal force acting on aparticle is known to be equal to:

$$f = pw^2 m \qquad (2.6)$$

p: perpendicular distance of the particle from the spin axis
w: earth's spin angular velocity
m: mass of the particle.

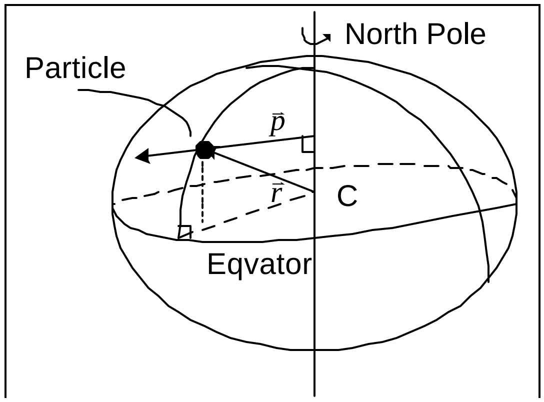


Figure 2.4 Centrifugal Force

If $w = 72.921151467 * 10^{-6}$ rad/s
$p = 6378.160$ km

The value of centrifugal force on the equator

$f = 3{,}392\ cm/s^2 * m$

which is about 0.35% of the gravitational force. On the poles the centrifugal force vanishes.

The sum of the gravitational force and the centrifugal forces is called the gravity force. The field of this force is shown in Figure 2.5 by bold arrows. Gravity force is stronger on the Poles than on the equator. The difference would be about 0.35%, If the earth were spherical. Since the earth is oblate, the difference is even more pronounced (0.54%).

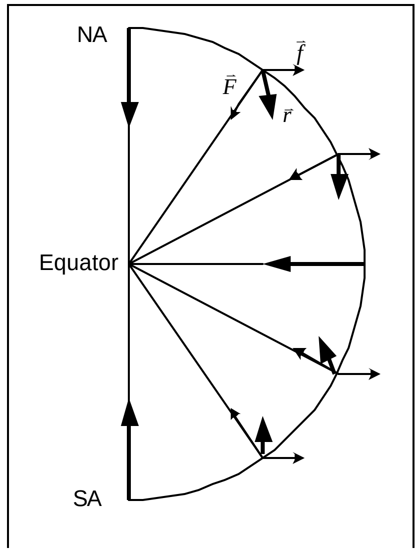


Figure 2.5. The gravity force

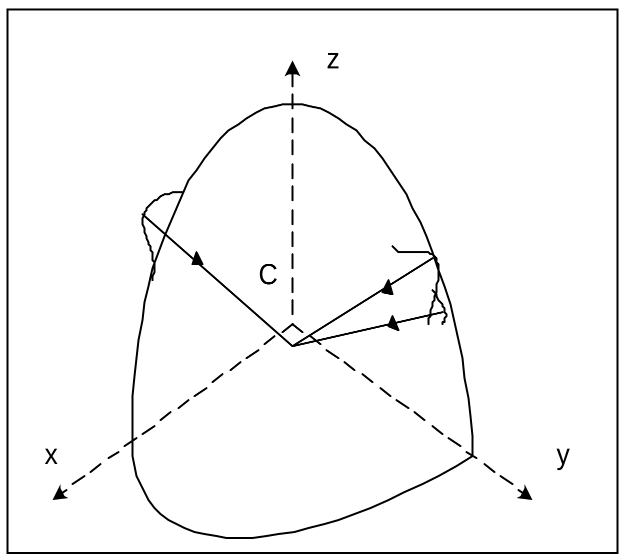


Figure 2.6. Direction of gravity

It is usual to work with accelerations rather than forces. Summation of eqn 4 and 5

From Newton's second law, force is the product of acceleration and mass. The term in brackets must be the vector of acceleration and denoted by g and is called the gravity vector.

$$\vec{F}'(\vec{r}_A) = \vec{F}_{B\to A} * \vec{f}_A = \left\{ G\iiint_B \frac{s(\vec{r})}{|\vec{r}-\vec{r}_A|^3}(\vec{r}-\vec{r}_A)\,dB + \vec{p}_A w^2 \right\} m \qquad (2.7)$$

The gravity field g gives the complete geometrical picture of the gravity force field.

$$\vec{F}'(\vec{r}_A) = \vec{g}(\vec{r}_A)m \qquad (2.8)$$

The gravity field has a magnitude and direction (Figure 2.6). The magnitude is easier to deal. Units are Gal (Galileo Galilei)

$$1 \text{ Gal} = 1 \text{ cm/s}^2$$

The mean magnitude of gravity on the surface of the earth is of the order of 980.3 Gal.

2.2. Gravity Anomaly

The magnitude of gravity g can be measured using gravity measuring instruments. From the measurements gathered from all over the world, the magnitudes vary globally and regionally as well as locally. The global range of variations on the surface of the earth is more than 5 Gal (more than 0.5% of averge g). Modern measurements can measure accurately up to 10^{-10} g. The variations are due to:

- different heights of observation points
- the oblateness of the earth
- the uneven lateral distribution of masses within the earth

a) gravity variation with height:

$$g = G\frac{M}{r^2} \quad \text{(approved from Eq. 1)} \qquad (2.10)$$

$$\frac{dg}{dr} = -2\frac{GM}{r^3} \quad \text{r: distance from the earth center} \qquad (2.11)$$

$dr \cong dH$ gravity gradient in radial direction $\rightarrow$ $dg = -2\frac{GM}{r^3}dH$ (2.12)

GM= 3.98603×10^{20} cm^3/s^2, r=6371.031 km

$$dg = -0.308\left[mGal m^{-1}\right]dH \qquad (2.13)$$

The gravity increment with height at or near the surface of the earth. With increasing height gravity magnitude decreases. Gravity decreases only by about 0.36% for the top of Everest.

$dg \downarrow$ 1% with $H \uparrow$ 32 km

The height effect correction is called as the *free air correction*.

b) The most widely used technique is to first correct the observed gravity for the height effect, and then compared with analytically defined *reference gravity*. Consideringn a massive biaxial (ellipsoid), concentric with the earth, the minor axis coincides with the polar principle axis of inertia of the earth. (it spins with w)

Such a reference gravity field is called the normal gravity field and it is represented by the normal gravity vector, The normal field is a function of both the distance from the centre of mass of the earth and lattitude ϕ. There are some formulas giving the normal gravity (refering as the years). The difference btw the reduced actual gravity and normal gravity on the ellipsoid is called the gravity anomally and is denoted by Δg.

International gravity formula (1967)

$$g_0 = 978.03185\left(1 + 0.005278855\ \sin^2 f + 0.000023464\ \sin^4 f\right)$$

c) Gravity variations due to the irregular distribution of masses within the earth.

$g > g$ Σ there are denser masses
$g < g$ Σ negative density anomally

2.3. Gravity Potential

Gravity force field is irrotational, therefore it has a potential energy.

$$\vec{F} = m\vec{g} = \nabla V = m\nabla w \qquad (2.14)$$

$$\vec{g} = \nabla w \qquad (2.15)$$

This scalar field is gravity potential.

$$\vec{g} = \vec{g}_g + \vec{g}_c = \nabla w_g + \nabla w_c = \nabla(w_g + w_c) \qquad (2.16)$$

$$w_g(\vec{r}_A) = G\iiint_B \frac{s\,(\vec{r}\,)}{(\vec{r} - \vec{r}_A)}dB \qquad (2.17)$$

$$w_c(\vec{r}_A) = \frac{1}{2}p_A{}^2 w^2 \qquad (2.18)$$

Wc increases proportionally to the square of the distance Pa from the spin axis. While Wg decreases above the earth for $|\vec{r}_A| > |\vec{r}|$ being inversely proportional to the distance $|\vec{r} - \vec{r}_A|$ (Figure 2.7)

The gravity equipotential surface is a surface on which the gravity potential is constant.

W(r) = constant

Infinite number of equipotential surfaces can be found just by assigning different values to the potential. The lines of force are the curves to which the gradient of the potential is tengent at every point. The lines of force of the earth's gravity field are called the **PLUMB LINES** (Figure 2.8).

The equipotential surface which coincides with the mean sea level over the earth is called the **geoid**.

When moving along an equipotential surface, no change in the potential is experienced and no work, in static sense , is done, either positive or negative.

If a cross section of an equipotential surface is drawn (Figure 2.5) it makes an oblate curve. All the equipotential surfaces make an oblate spatial pattern (Figure 2.8) of a series of concentric ellipsoids. The direction of gravity and equipotential surfaces are perpendicular.

- What is the relation btw the equipotential surfaces and the magnitude of gravity?

The closer together the surfaces, the stronger the gravity field (Figure 2.9).

$$g = |\nabla w| = -\frac{dw}{dh} \tag{2.19}$$

- Is gravity on an equipotential surface constant?

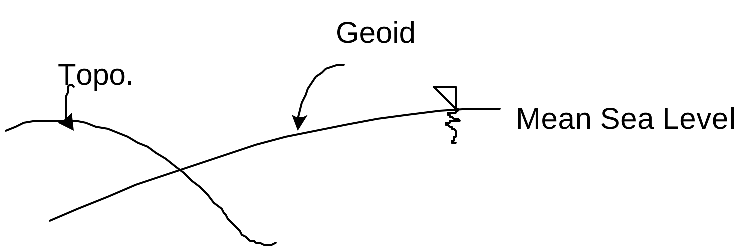
The magnitude of gravity on an equipotential surface varies.

Gravity is stronger on poles than on the equator.

From the Int. Gravity Formula (1967)
More work is needed to lift a body of constant mass at the pole than it is at the equator.
The surfaces of the lakes and oceans tend to follow the gravity equipotential surfaces with only minor deviations.

2.4. Geoid and deflections of the vertical

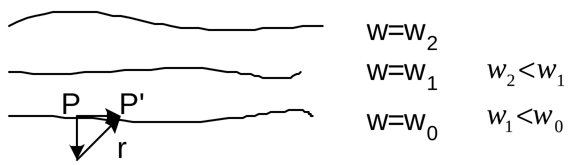
The one gravity potential surface of particular interest is that which best approximates the mean sea level over the whole earth. It is called the geoid.



Geoid is playing fundamental role in positioning.

The equation of geoid surface is:

$w=w_0=6.2368085*10^7\ m^2/s^2$ called geopential constant



Consider two close points P (u_1,u_2,u_3) and P’ (u_1+du_1, u_2+du_2, u_3+du_3) on the same equipotential surface $W = W_0$

$$w(p)=w(u_1,u_2,u_3)$$
$$w(p')=w(u_1+du_1,u_2+du_2,u_3+du_3)$$
$$w(p')=w(p)=w(u_1,u_2,u_3)+\frac{dw}{du_1}du_1+\frac{dw}{du_2}du_2+\frac{dw}{du_3}du_3=w_0$$
$$w(p')=\left(\frac{dw}{du_1}\underline{i}+\frac{dw}{du_2}\underline{j}+\frac{dw}{du_3}\underline{k}\right)*\left(du_1\underline{i}+du_2\underline{j}+du_3\underline{k}\right)=0$$
$$0=\nabla w*dr$$
$$0=g*dr$$

Gravity vector is perpendicular to dr which is tangent to equipotential surface.

Observations have shown that the geoid can also be approximated up to some tens of meters- by a biaxial geocentric ellipsoid ahose minor axis coincides with the earth’s principal polar axis of inertia. The best fitting geocentric ellipsoid is called as the mean earth ellipsoid (Figure 2.10).

Actual gravity field	normal gravity field
Actual equipotential surfaces	normal equipotential surfaces
Actual plumb line	normal plumb line
Geoid	geocentric ellipsoid

Separation between the geocentric reference ellipsoid and the geoid is called the absolute geoidal height.
Relative geoidal height $\rightarrow$ refers the geoid to another kind of reference ellipsoid that is not geocentrically located.

In Figure 2.11 Absolute geoid for Turkey is given.

The actual and the normal gravity vactors are given in Figure 2.12 to show the irregularities of the earth’s gravity field.

The spatial angle between g_s and g_0 defines the deflection of the vertical q. The angle of interest between the actual gravity vector taken on the earth’s surface and the ellipsoidal normal is called the surface deflection. It can be either absolute or relative according to which kind of ellipsoid is used (geocentric or non-geocentric). The differences are expected to be more significant in mountainous regions.

The deflections can be regarded as being composed of two parts:
One reflecting the regional density distribution (predominate in flat and low areas)
Complexity of the earth's surface topography (in mountanious releiefs).

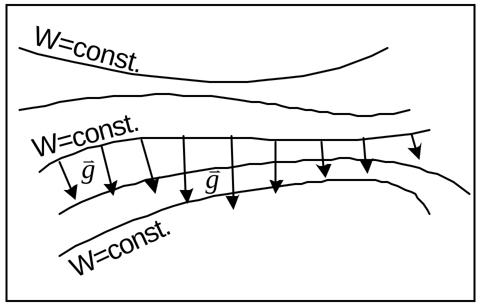
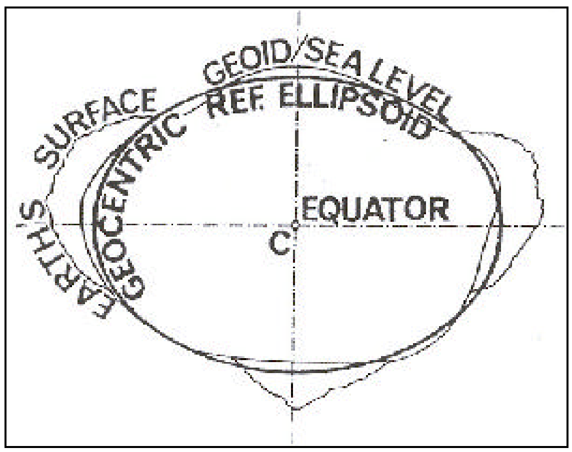
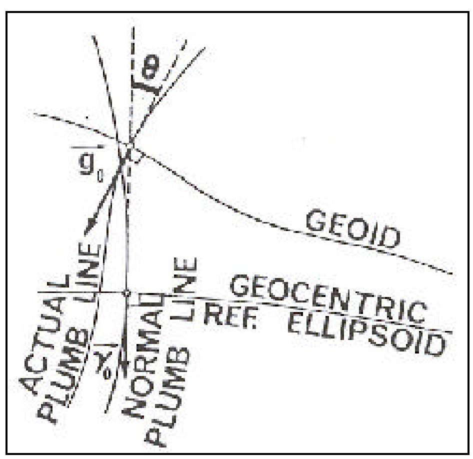


Figure 2.9 Gravity on an equipotential surface.



2.10 Biaxial ellipsoid as a normal body of the Earth



2.11 Deflection of the vertical