An Interesting Number in Physics

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Physical constants play important role in physics. It is a fact that the accuracy of the physical constants grows year by year. Special attention is paid to the dimensionless constants; the most familiar among them are the fine structure constant, the proton/electron mass-ratio, the cosmological constant of Einstein's general relativity, the Weinberg angle in the electro-weak interaction theory, *etc*. One of the most important questions has for a long time been: are there any physical and/or mathematical relations between the fundamental physical constants? This paper gives a recently explored simple math relation between them. The precise theoretical explanation for this amazing finding needs more detailed investigation related to the physical background.
Keywords: physical constants, exponential relation between physical constants, background of the physical constants, Bode-Titius law, neutral-atom mass formula.

1. Introduction

In Nature, an exponential dependence between the observable quantities frequently occurs, without causing any surprise for us. Typical examples are the radioactive decay in physics, or the bacteria propagation in biology. The speed distribution of molecules shows exponential function in the Maxwell-Boltzmann kinetic theory of the gases. Do there exist any exponential relations between the fundamental physical constants? This was the question that motivated the present paper. The statistical studies undertaken led to an amazing result: the fundamental physical constants are connecting to each other with a simple exponential form. For example, here is an expression for the dimensionless fine structure constant:

$$\alpha^2 / 2 \cong Q^7 \tag{1.1}$$

where the exponentiated number is:

$$Q \equiv 2/9 \equiv 0.222... \tag{1.2}$$

A similar finding is:

$$m_e / M_p \cong Q^5 \tag{1.3}$$

where m_e is the electron mass, M_p is the proton mass. A third example shows exponential relation between the electron mass and muon mass:

$$m_e / 2m_\mu \cong Q^4 \tag{1.4}$$

In this paper it will be clearly shown that the number $Q = 2/9$ has a central significance in the mathematical connection between the fundamental physical constants. The constants explored and results demonstrated lead us to believe that behind of these exponential connections of fundamental physical constants there must lie an important physical background. Nevertheless, the exact physical background is missing at present.

2. Exponential Forms for the Fundamental Physical Constants

The exponential form introduced above is approximately valid for many of the dimensioned fundamental physical constants, which are expressed in the internationally accepted and applied SI units. Generally, a physical constant X can be written into a simple mathematical expression:

$$\lambda X \cong Q^S; \quad Q \equiv 2/9 \tag{2.1}$$

where S is approximately an integer, and λ is a 'simple' constant like the '1/2' in (1.1) or (1.4).

Table 1. Q -Forms of important physical constants

Physical Constant (SI)	Q -Form	S(calc.)	S(int.)
Speed of light	c	-12.977125	-13
Gravitational const.	$G/2$	16.038625	16
Coulomb const.	$\pi\kappa$	-15.999071	-16
Elementary charge	$e/\sqrt{2}$	29.004044	29
Planck const.	$\hbar$	52.015115	52
Boltzmann const.	k	34.996134	35
Rydberg const.	$\kappa e^2/2R_B$	27.038014	27
Bohr radius	πR_B	14.971007	15
Electron mass	m_e	45.988879	46
Muon mass	$2m_\mu$	41.983270	42
Tau-particle mass	$m_\tau/2$	41.028418	41
π_0 mass	$\pi_0/3$	43.011789	43
$\pi_\pm$ mass	$\pi_\pm/3$	42.989518	43
Proton mass	M_p	40.992176	41
Neutron mass	M_n	40.991249	41

Table 1 shows the results obtained regarding the most important physical constants. The physical constants are expressed in SI units, with values obtained from the database of the National Institute of Standards and Technology [1].

3. Exponential Interpretation of the Bode-Titius Law

Bode's law, better called the Bode-Titius Rule, was first published by Johann Daniel Titius, but did not become well known until it was republished by Johann Elert Bode in the 18th century. It is supposed to predict the distances of the planets from the Sun in astronomical units (Sun-Earth middle distance) by the formula $0.4 + 0.3 \times 2^n$, but is usually represented by a Table, shown here as Table 2.

Table 2. Demonstration of the Bode-Titius Rule.

Planet	Calculation	Predicted	Measured
Mercury	$0.4 + 0.3 \times 0$	0.4	0.39
Venus	$0.4 + 0.3 \times 1$	0.7	0.72
Earth	$0.4 + 0.3 \times 2$	1.0	1.00
Mars	$0.4 + 0.3 \times 4$	1.6	1.52
Ceres	$0.4 + 0.3 \times 8$	2.8	2.77
Jupiter	$0.4 + 0.3 \times 16$	5.2	5.20
Saturn	$0.4 + 0.3 \times 32$	10.0	9.54
Uranus	$0.4 + 0.3 \times 64$	19.6	19.19
Neptune	$0.4 + 0.3 \times 128$	38.8	30.07

The second column in Table 2 gives the formula for the distance to each planet, and the third column gives the result of that calculation. The fourth column shows the actual average distance from the Sun for each planet. Ceres was discovered by chance, not by application of the Bode-Titius rule. Nevertheless, its orbit fit the rule so perfectly that there had been active search for a planet at that distance, and the discovery was considered to be another vindication. The Bode-Titius rule was used in the calculations that led to the discovery of Neptune. It is remarkable that the physical background of this observed rule, which shows at least exponential behavior of the planet distances from the Sun, has remained unclear until this time.

In the framework of present study, the Bode-Titius rule has been fitted to the recognized exponential relation involving the 'special number' Q . The simple expression of the Kepler's third law is:

$$P^2 / a^3 = \text{constant} \quad , \tag{3.1}$$

where P is the orbital period and a is the semi-major axis of the orbit for the planets of Solar System. When certain units are chosen, namely P is measured in sidereal years and a in astronomical units, P^2 / a^3 has the value 1 for all planets in the Solar System. For this reason Kepler's third law for the planets can be written into simple form:

$$P_n^2 / a_n^3 \equiv Q^n / Q^n \cong 1; \quad (n = \text{integer}) \tag{3.2}$$

where for the Earth the selection $n = 0$ is valid.

This approximation defines the astronomical distance of each planet from the Sun in exponential form:

$$a_n \cong Q^{n/3} \quad ; \quad (n = \text{integer}) \tag{3.3}$$

Nevertheless, in this equation the number Q does not have a fixed value. Table 3 shows the calculated different Q -values for the actual distances of each planet from Eq. (3.3):

Table 3. Results of the Q -calculations.

Planet	a_n	*n*	*Q (calc.)*
Mercury	0.39	2	0.243555
Venus	0.72	1	0.373248
Earth	1	0	--------
Mars	1.52	-1	0.284754
Ceres	2.77	-2	0.216910
Jupiter	5.20	-3	0.192308
Saturn	9.54	-4	0.184220
Uranus	19.19	-5	0.169885
Neptune	30.07	-6	0.182362

The average of the calculated Q -values is near to its 'nominal value' 2/9:

$$\langle Q \rangle = 0.230905... \cong 2/\ 9 \equiv 0.222... \tag{3.4}$$

The standard deviation of the calculated Q -values is:

$$\sigma(Q) \cong 11\% \tag{3.5}$$

This interesting result strengthens the supposed physical significance of the explored special number Q .

4. The Weak Mixing Angle

The weak mixing angle or Weinberg angle is a parameter in the *Weinberg-Salam* theory of the *electroweak force.* It gives a relationship between the $W_\pm$ and Z-masses. The mixing angle weakly depends on the momentum transfer in the particle accelerators. If the momentum transfer corresponding to the mass creation of the Z_0 boson (*91.2 GeV/c*), the experimentally best estimated value of the Weinberg parameter is:

$$\sin^2 \theta_W = 0.23120(15) \cong Q \quad . \tag{4.1}$$

where θ_W is the Weinberg parameter. Its value $0.23120(15)$ is practically equal to that average value of Q that was obtained from the above-described exponential model of the planet distances in the Solar System. Can this fact be only blind chance? Now, of course, the question is open.

5. The *Q* Number in the Neutral-Atom Mass Formula

In an earlier paper published in Galilean Electrodynamics [2], a new atomic mass formula was given for the neutral atoms. The background of this formula is a special application of Planck's radiation law. Expressed in atomic units, the mass of a neutral atom having mass number A, is approximately:

$$M(A) \cong AM_0\left[1-\tfrac{1}{2}m_e(A-6)\Big/M_p\left(\eta^{\sqrt{A-2}}-1\right)\right] \;;\; (A\geq 3) \qquad (5.1)$$

where m_e is the electron mass, M_p is the proton mass, and M_0 and η are fitting parameters. The optimized fitting parameters are:

$$M_0 = 1.003304\,\text{a.u.} \;;\; \eta = 1.220127 \cong 1+Q \qquad (5.2)$$

The relative standard deviation of this simple formula, calculated from a large amount of experimental data is:

$$\sigma = 3.216\times 10^{-4} \qquad (5.3)$$

Taking account of the experience of (1.3) related to the special number Q, the mass formula (5.1) can be rewritten into the expression

$$M(A) \cong AM_0\left\{1-\tfrac{1}{2}Q^5(A-6)\Big/\left[(1+Q)^{\sqrt{A-2}}-1\right]\right\} \;;\; (A\geq 3) \qquad (5.4)$$

where the new fitting parameters are M_0 and Q. The fitting of the (5.4) modified mass formula has been updated with nearly 2000 measured neutral atomic masses obtained from the publication of G. Audi and A.H. Wapstra [3]. The results of the new calculation are:

$$M_0 = 1.003393\,\text{a.u.} \;;\; Q = 0.226266 \;;\; \sigma = 3.186\times 10^{-4} \qquad (5.5)$$

Finally, here is the extended neutral-atom mass formula containing the Z-dependence of the atoms (in atomic unit):

$$M(A,Z) \cong M_0\left\{A-\tfrac{1}{2}Q^5(A-1)(A-3)\Big/\left[\left(1+Q\right)^{\sqrt{A-1}}-1\right]\right\}+M_a+M_s \qquad (5.6)$$
$$(A\geq 3)$$

where M_a is called *the asymmetric mass correction* and M_s is called *the symmetric mass correction*:

$$M_a = C_a(A/2-Z)^2\Big/(A+6)^2 \;;\; M_s = -C_s\delta(Z,N)\Big/\sqrt{A} \qquad (5.7)$$

The function $\delta(Z,N)$ is well known from Weizsäcker's formula applied for the nuclei [4]:

$$\delta(Z,N) = \begin{cases} 1 & (Z=\text{even}, N=\text{even}) \\ 0 & (Z+N=\text{odd}) \\ -1 & (Z=\text{odd}, N=\text{odd}) \end{cases} \qquad (5.8)$$

It is important to mention that the given Z-corrections were *empirically determined* in the fitting procedures aiming to reach the best accuracy. The optimized fitting parameters and the relative standard deviation of the mass values are the next:

$$M_0 = 1.003272\,\text{a.u.} \;;$$
$$Q = 0.224584 \;;\; \sigma = 1.313\times 10^{-4} \;; \qquad (5.9)$$
$$C_a = 1.375266 \;;\; C_s = 7.627297\times 10^{-3} \;.$$

The Z-dependent formulation for the masses of neutral atoms gives remarkably better accuracy. The obtained important parameter Q is in good agreement with its 'nominal value' 2/9.

6. Conclusions

This paper has introduced a special number, $Q = 2/9$, which is suitable to express many important physical constants in similar exponential forms. From the illustrative examples one can safely conclude that the fundamental physical constants are very likely quantified by a special exponential rule. It follows directly from this statement that there must exist among of all fundamental physical constants a simple exponential relationship. The next important question is whether this statement is an axiom, without any possibility for a deeper physical explanation, or must it have an unknown physical background. To answer this important question, more detailed research is certainly needed in the future.

References

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