On the Capacity of UWB over Multipath Channels

Fernando Ramírez-Mireles, *Senior Member, IEEE*

***Abstract*— In this work we compute the information theoretic capacity C of binary orthogonal pulse position modulated (PPM) signals for ultra wideband (UWB) communications over multipath channels. We consider binary PPM signals with random energy and correlation values. Numerical examples are given to illustrate the capacity results.**

***Index Terms*— Ultra wideband communications, pulse position modulation, channel capacity, multipath channels.**

I. Introduction

COMMUNICATIONS using UWB with time hopping (TH) and PPM has been studied extensively [1]- [3]. It also has been proposed for consideration in IEEE standard bodies [4]. This work calculates C for orthogonal UWB PPM signals over multipath channels considering the random variations in both the received signal energy and the signal correlation values.

In contrast, previous work calculated C for the additive white Gaussian noise (AWGN) channel [5] [6], and the work in [7] studied C over multipath channels considering only the random variations in the received signal energy.

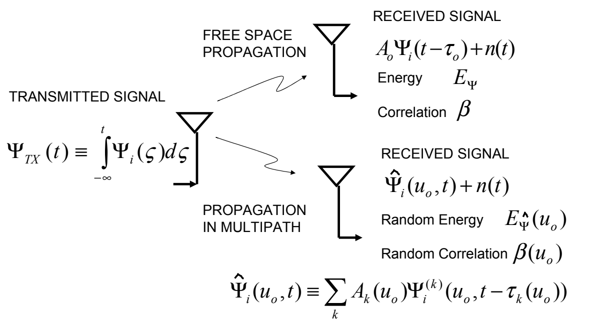


Fig. 1. Signals and parameters considered for capacity calculation.

II. System Model

We assume detection using a receiver perfectly synchronized and matched to the received signals, with time-invariant channel conditions valid during a bit interval. Fig. 1 shows the situation considered in this work.

Manuscript received October 22, 2004. The associate editor coordinating the review of this letter and approving it for publication was Prof. Gianluca Mazzini.

The author is with the Instituto Tecnológico Autónomo de México (ITAM), Ciudad de México, D.F. C.P. 01000, México (e-mail: ramirezm@ieee.org).

Digital Object Identifier 10.1109/LCOMM.2005.06028.

Under free space propagation conditions (i.e., the AWGN channel with $n(t)$ having two-sided power spectrum density $N_o/2$), the received signal $\Psi_i(t)$, $i = 1, 2$, is the derivative of the transmitted signal Ψ_{TX} [1], modified by amplitude A_o and delay τ_o factors that depend on the transmitter-receiver separation distance D,[1] where $E_\Psi \triangleq \int_{-\infty}^{\infty} [\Psi_i(t)]^2\, dt$ is the received signal energy and $\beta \triangleq \frac{1}{E_\Psi}\int_{-\infty}^{\infty} \Psi_1(t)\Psi_2(t)dt$ is the normalized correlation value between $\Psi_1(t)$ and $\Psi_2(t)$, with $\beta = 0$ for orthogonal signals. In this case C depends solely on the signal-to-noise ratio (SNR) [8].

Under multipath conditions (e.g., a slowly varying indoor radio channel, where the transmitter is placed at a certain fixed location, and the receiver is placed at a variable location denoted u_o), the received signal $\hat{\Psi}_i(u_o, t)$ is a multipath spreaded version consisting of multiple replicas $\Psi_i^{(k)}(u_o, t)$, each one with different amplitude $A_k(u_o)$, delay $\tau_k(u_o)$, and frequency content [9], where $E_{\hat{\Psi}}(u_o) \triangleq \int_{-\infty}^{\infty}[\hat{\Psi}_i(u_o, \xi)]^2 d\xi$ is the random signal energy, and where $\beta(u_o) \triangleq \frac{\int_{-\infty}^{\infty} \hat{\Psi}_1(u_o,\xi)\, \hat{\Psi}_2(u_o,\xi)\, d\xi}{E_{\hat{\Psi}}(u_o)}$ is the random normalized correlation value between $\hat{\Psi}_1(u_o, t)$ and $\hat{\Psi}_2(u_o, t)$. Even when $\beta = 0$, $\beta(u_o)$ is a random variable that is not necessarily zero [10]. In this case $C(u_o)$ is also dependant on the value u_o (i.e., depends on both $E_{\hat{\Psi}}(u_o)$ and $\beta(u_o)$), and the average capacity is obtained by taking the expected value $\overline{C} \triangleq \mathbf{E}_u\{C(u)\}$ of $C(u_o)$ over the multipath effects.

A. PPM Signals over the Gaussian Channel

The binary orthogonal PPM signals considered here are [2]

$$\Psi_i(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - (i-1)\delta), \quad i = 1, 2. \tag{1}$$

The T_f is the frame repetition period. The duration of $\Psi_i(t)$ is $T_s = N_sT_f$. The signal $w(t)$ is the basic UWB pulse used to convey information. It has duration T_w and energy $E_w = \int_{-\infty}^{\infty} [w(t)]^2\, dt$. The normalized signal correlation function of $w(t)$ is $\gamma(\tau) \triangleq \frac{1}{E_w}\int_{-\infty}^{\infty} w(t)w(t-\tau)dt > -1\,\forall\tau$. By assuming $T_f > T_w + \delta$ we get that $\Psi_i(t)$ in (1) have $E_\Psi = N_sE_w$ and also that $\beta = \gamma(\delta)$. For $\delta \geq T_w$ the signals become orthogonal with $\beta = 0$.

B. PPM Signals over the Multipath Channel

The transmitted pulse is the same pulse $w_{\text{TX}}(t) \triangleq \int_{-\infty}^{t} w(\xi)d\xi$ used in the Gaussian channel case, and the received "pulse" is $\sqrt{E_a}\hat{w}(u_o, t)$. The $\sqrt{E_a}\hat{w}(u_o, t)$

[1] In our analysis we will assume $A_o = 1$ and $\tau_o = 0$.

is a multipath spread version of $w(t)$ received at position u_o, it has average duration $T_a >> T_w$ and random energy $E_{\hat{w}}(u_o) \triangleq E_a \alpha^2(u_o)$, where E_a is the average energy and $\alpha^2(u_o) \triangleq \int_{-\infty}^{\infty}[\hat{w}(u_o,t)]^2 dt$. The $\sqrt{E_a}\hat{w}(u_o,t)$ has random correlation function $\gamma(u_o,\tau) \triangleq \frac{\int_{-\infty}^{\infty}\sqrt{E_a}\hat{w}(u_o,t)\sqrt{E_a}\hat{w}(u_o,t-\tau)\,dt}{E_{\hat{w}}(u_o)} > -1\ \forall\tau$. Clearly, the multipath effects change for different u_o, and therefore $E_{\hat{w}}(u_o)$ and $\gamma(u_o,\tau)$ both change with u_o [10].

Using the previous pulse definition, the PPM signals containing the multipath effects can be denoted

$$\hat{\Psi}_i(u_o,t) = \sum_{k=0}^{N_s-1}\sqrt{E_a}\hat{w}(u_o,t-kT_f-(i-1)\delta),\ i=1,2. \quad (2)$$

For simplicity we assume that $\hat{\Psi}_i(u_o,t)$ has fixed duration $T_s \simeq N_s T_f$. By assuming $T_f > T_a+\delta$ we get that $\hat{\Psi}_i(u_o,t)$ in (2) have $E_{\hat{\Psi}}(u_o) \simeq \overline{E}_\Psi \alpha^2(u_o)$, where $\overline{E}_\Psi = N_s E_a$ is the average signal energy, and also that the $\hat{\Psi}_1(u_o,t)$ and $\hat{\Psi}_2(u_o,t)$ have $\beta(u_o) \simeq \gamma(u_o,\delta)$. Even though $\beta = 0$ for $\delta \geq T_w$, $\beta(u_o)$ is a random variable with values in $(-1,1)$.

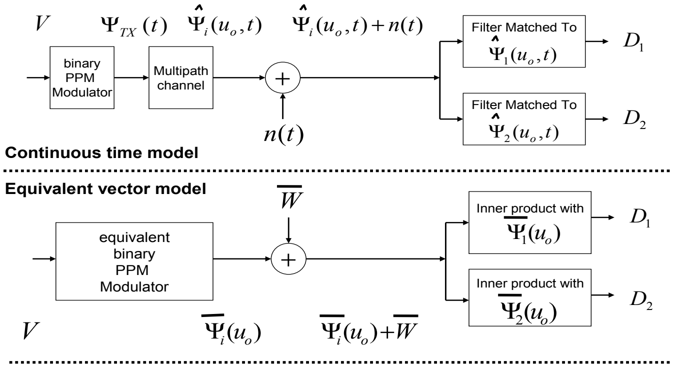
C. The Choice of δ and T_f

In a "typical" UWB TH PPM system design [1]- [3], δ lasts a few ns (similar to T_w), T_f lasts a few hundred of ns, and N_s has a value of dozens or even a few hundred. The symbol rate is $R_s = \frac{1}{T_f N_s}$. In a multi-user environment with TH present, the pulse hops over a range $T_f - \delta - T_w$, and the processing gain is $G \simeq N_s \frac{T_f}{T_w}$.

In this work we use $\delta = T_w$. To avoid interpulse interference we use $T_f > T_a + \delta$ (for consistency, this condition is applied to both Gaussian and multipath channels). This assumption simplifies the calculation of $\beta(u_o)$ for $\hat{\Psi}_i(u_o,t)$ to get a value approximately similar to $\gamma(u_o,T_w)$ for $\hat{w}(u_o,t)$. Even if interpulse interference were present, intersymbol interference could be neglected for large N_s.

D. Vector Model for Capacity Calculations

Capacity calculations are based on energy and correlation values of the *received* signals. We consider, for the time being, that the multipath conditions are being kept fixed, i.e., the capacity calculations are conditioned on a particular value u_o. To calculate the capacity we use the channel model in Fig. 2. We consider a binary PPM modulator with input V and output $\overline{\Psi}_i(u_o)$, $i=1,2$. The scalar V is the output of a 1-bit source, with zeros and ones being equally likely. The 2-dimensional vector $\overline{\Psi}_1(u_o) \triangleq \sqrt{E_\Psi(u_o)}\,(\psi_1(u_o), +\psi_2(u_o))$ represents the UWB signal $\hat{\Psi}_1(u_o,t)$, and the vector $\overline{\Psi}_2(u_o) \triangleq \sqrt{E_\Psi(u_o)}\,(\psi_1(u_o), -\psi_2(u_o))$ represents $\hat{\Psi}_2(u_o,t)$, where $\psi_1(u_o) \triangleq \sqrt{\frac{1+\gamma(u_o,\delta)}{2}}$ and $\psi_2(u_o) \triangleq \sqrt{\frac{1-\gamma(u_o,\delta)}{2}}$. These vectors are the projections of the received signals with respect to an orthonormal basis whose elements are $\Phi_1(u_o,t) \triangleq \frac{\hat{\Psi}_1(u_o,t)+\hat{\Psi}_2(u_o,t)}{2\sqrt{E_{\hat{\Psi}}(u_o)}\,\psi_1(u_o)}$ and $\Phi_2(u_o,t) \triangleq \frac{\hat{\Psi}_1(u_o,t)-\hat{\Psi}_2(u_o,t)}{2\sqrt{E_{\hat{\Psi}}(u_o)}\,\psi_2(u_o)}$. The reader can verify that $\overline{\Psi}_1(u_o)$, $\overline{\Psi}_2(u_o)$ indeed have energy $E_\Psi(u_o)$ and correlation value $\beta(u_o) = \gamma(u_o,\delta)$.



$$D_j = <\hat{\Psi}_i(u_o,t), \hat{\Psi}_j(u_o,t)> + <n(t), \hat{\Psi}_j(u_o,t)>$$
$$= <\overline{\Psi}_i(u_o), \overline{\Psi}_j(u_o)> + <\overline{W}, \overline{\Psi}_j(u_o)>;\ j=1,2.$$

Fig. 2. Continuous channel model and equivalent vector channel model used for capacity calculations, where the inner product between signals is defined as $<\hat{\Psi}_1(u_o,t), \hat{\Psi}_2(u_o,t)> \triangleq \int_0^{T_s}\hat{\Psi}_1(u_o,t)\hat{\Psi}_2(u_o,t)\,dt$, and between vectors as $<\overline{\Psi}_1(u_o), \overline{\Psi}_2(u_o)> \triangleq \overline{\Psi}_1(u_o)\cdot\overline{\Psi}_2(u_o)$.

The vector $\overline{\Psi}_i(u_o)$ is sent through the vector Gaussian channel in Fig. 2. The output of the channel is $\overline{Y}(u_o) = \overline{\Psi}_i(u_o) + \overline{W}$, where $\overline{Y}(u_o) \triangleq (y_1(u_o), y_2(u_o))$, and $\overline{W} \triangleq (\varrho_1, \varrho_2)$ is a real Gaussian noise vector with zero mean and variance $\sigma^2 = \frac{N_o}{2}$ in each dimension.

III. Calculation of Channel Capacity

In this section we calculate the information-theoretic channel capacity for the binary PPM vectors $\overline{\Psi}_i(u_o)$. The capacity derivation generalizes the calculations done for orthogonal signals in [8]. The channel capacity with input signals restricted to a discrete set of binary equally-likely non-orthogonal PPM signals, and continuous-valued outputs, can be found to be

$$\begin{aligned} C(u_o) = 1 & \\ & -\frac{1}{2}\mathbf{E}_{\overline{Y}(u_o)|\overline{\Psi}_1(u_o)}\left\{\log_2\left(1+\frac{p(\overline{Y}(u_o)|\overline{\Psi}_2(u_o))}{p(\overline{Y}(u_o)|\overline{\Psi}_1(u_o))}\right)\right\} \\ & -\frac{1}{2}\mathbf{E}_{\overline{Y}(u_o)|\overline{\Psi}_2(u_o)}\left\{\log_2\left(\frac{p(\overline{Y}(u_o)|\overline{\Psi}_1(u_o))}{p(\overline{Y}(u_o)|\overline{\Psi}_2(u_o))}+1\right)\right\}, \\ & = 1 \\ -\frac{1}{2}&\mathbf{E}\left\{\log_2\left(1+\exp\left[-\frac{\sqrt{E_{\hat{\Psi}}(u_o)}\psi_2(u_o)}{(N_o/2)}y_2(u_o)\right]\right)\right\} \\ -\frac{1}{2}&\mathbf{E}\left\{\log_2\left(1+\exp\left[+\frac{\sqrt{E_{\hat{\Psi}}(u_o)}\psi_2(u_o)}{(N_o/2)}y_2(u_o)\right]\right)\right\}, \quad (3) \end{aligned}$$

where $p(\overline{Y}(u_o)|\overline{\Psi}_i(u_o))$ is the probability density function (p.d.f.) of $\overline{Y}(u_o)$ conditioned on $\overline{\Psi}_i(u_o)$, and $\mathbf{E}\{\cdot\} = \mathbf{E}_{\overline{Y}(u_o)|\overline{\Psi}_i(u_o)}\{\cdot\}$ is the expected value with respect to $\overline{Y}(u_o)$ conditioned on $\overline{\Psi}_i(u_o)$.

To calculate the capacity $C(u_o)$ in (3) we need to calculate the expectations $\mathbf{E}_{\overline{Y}(u_o)|\overline{\Psi}_i(u_o)}\{\cdot\}$. These expectations can be estimated via Monte Carlo simulation [8]. The method is to generate pseudorandom 2-dimensional vectors $\overline{Y}(u_o)$ according to the p.d.f. $p(\overline{Y}(u_o)|\overline{\Psi}_i(u_o))$. For each generated sample $\overline{Y}(u_o)$ the function inside the logarithm is evaluated. Finally the sample average of the logarithm is calculated.

To calculate the average capacity $\overline{C}$ we need to calculate the expectation $\mathbf{E}_u\{\cdot\}$. This expectation can also be estimated via Monte Carlo simulations using the sample mean value

$$\overline{C} \approx \frac{1}{u_*}\sum_{u_o=1}^{u_*} C(u_o). \tag{4}$$

computed using an ensemble of UWB pulse responses $\{\hat{w}(u_o,t)\}$, $u_o = 1,2,\ldots,u_*$. A detailed explanation of this method of performance calculation can be found in [11].

IV. Numerical Example

The UWB signals considered in this example are based on pulsed sine waves. The received pulse is modeled as $w(t) = \sin(2\pi\frac{10}{T_w}t)$, $0 \le t \le T_w$, with autocorrelation $\gamma(\tau) = \frac{1}{E_w}\frac{T_w-|\tau|}{T_w}\cos(2\pi\frac{10}{T_w}\tau)$, $-T_w \le \tau \le T_w$. The duration of $w(t)$ is $T_w = 2.0$ ns, with $E_w = \left(\frac{T_w}{2}\right)$. The spectrum of $w(t)$ is centered at $(\frac{10}{T_w}) = 5$ GHz, with a 10 dB bandwidth of about 700 MHz, satisfying the new definition of UWB signal stating that the 10 dB bandwidth of the signal should be at least 500 MHz [12].

To characterize the multipath channel we use an autoregressive channel model [13] [14] to form and ensemble of modeled channel pulse responses $\hat{w}(u_o,t)$ as described in [15]. We consider line-of-sight (LOS) scenarios with $D = 3,6,9$ m, and non-line-of-sight (NLOS) scenarios with $D = 1,2,3$ m. The simulated $\hat{w}(u_o,t)$ has $T_a \simeq 160$ ns. By selecting $T_f = 170$ ns we make sure that $T_f > T_a + \delta$.

A total of $u_* = 294$ channel pulse responses $\hat{w}(u_o,t)$ are used (49 per each distance value). An equal number of pairs $E_{\hat{\Psi}}(u_o), \beta(u_o)$ are calculated. These u_* sets of values are then used to compute (4). Fig. 3(a) show examples of energy values. Figs. 3(b) and 3(c) show examples of correlation values. Fig. 3(d) shows $\overline{C}$ vs. $E_b/N_o \triangleq \frac{\overline{E_\Psi/(N_o/2)}}{2C}$ for both AWGN and multipath channels. Compared with the AWGN channel, in the LOS multipath channel the 99 percent capacity is reached with an SNR disadvantage of about 4 dB. For the NLOS the SNR disadvantage is about 11 dB.

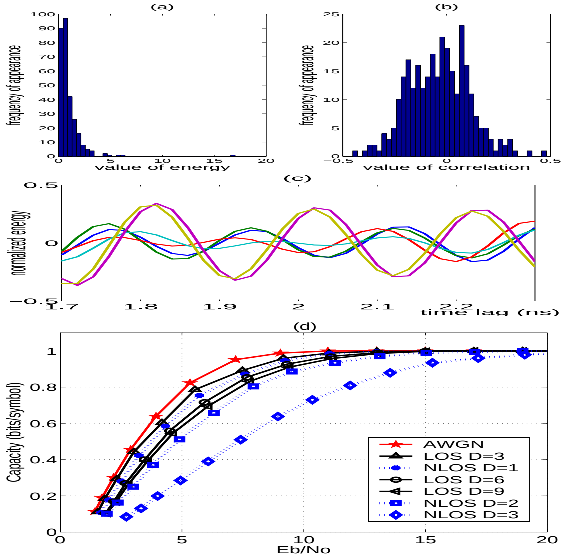


Fig. 3. (a) The histogram of $\alpha^2(u_o)$ for all distances considered (standard deviation 1.2674). (b) The histogram of $\beta(u_o)$ for all distances considered (standard deviation 0.1485). (c) Examples of $\gamma(u_o,\tau)$ for $1.7 \le \tau \le 2.4$ ns with random variations due to multipath. (d) Channel capacity results for AWGN and multipath (LOS and NLOS) channels.

V. Conclusions

This work computes the information theoretic capacity of binary orthogonal PPM signals for UWB communications over multipath channels. We consider binary PPM signals with random energy and correlation. A numerical example is given to illustrate the capacity results and quantify the SNR losses in the presence of multipath.

VI. Acknowledgments

The author thanks Mr. Enrique René Bastidas-Puga for providing the programs for the multipath channel model. He also thanks the constructive and enriching comments provided by the reviewers.

References

[1] R. A. Scholtz, “Multiple Access with Time Hopping Impulse Modulation,” invited paper, in *Proc. IEEE MILCOM Conf.*, pp. 447-450, 1993.
[2] F. Ramírez-Mireles and R. A. Scholtz, “System performance of impulse radio modulation,” in *Proc. IEEE RAWCON Conf.*, pp. 67-70, Aug. 1998.
[3] Special Issue on Ultra-Wideband Radio in Multiaccess Wireless Communications, *IEEE J. Sel. Areas Commun.*, vol. 20, Dec. 2002.
[4] A. F. Molisch *et. al.*, *Mitsubishi Electrics Time-Hopping Impulse Radio standards proposal*, IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs), Contribution IEEE P802.15-03113, May, 2003.
[5] L. Zhao and A. M. Haimovich, “Capacity of M-ary PPM ultra-wideband communications over AWGN channels,” in *Proc. IEEE Vehic. Tech. Conf.*, pp. 1191-1195, Oct. 2001.
[6] H. Zhang and T. A. Gulliver, “Performance and capacity of pulse position amplitude modulation in ultra-wideband communication systems,” in *Proc. IEEE Wireless Commun. and Networking Conf.*, pp. 895-900, Sept. 2003.
[7] Li Zhao, A. M. Haimovich, and M. Z. Win, ”Capacity of ultra-wide bandwidth communications over multipath channels,” in *Proc. IEEE Symp. on Advances in Wireless Commun. (ISWC'02)*, pp. 1-2, Sep. 2002.
[8] S. Dolinar *et. al.*, “Capacity of pulse-position modulation (PPM) on Gaussian and Webb channels,” *JPL TMO Progress Report,* vol. 42-142, pp. 1-31. Apr.-June 2000.
[9] R. C. Qiu and I.-T. Lu, “Multipath resolving with frequency dependence for wide-band wireless channel modeling,” *IEEE Trans. Veh. Technol.*, vol. 48, pp. 273-285, Jan. 1999.
[10] F. Ramírez-Mireles, “Signal design for ultra wideband communications in dense multipath,” *IEEE Trans. Veh. Technol.*, vol. 51, pp. 1517-1521, Nov. 2002.
[11] F. Ramírez-Mireles, “On performance of ultra wideband signals in Gaussian noise and dense multipath,” *IEEE Trans. Veh. Technol.*, vol. 50, pp. 244-249, Jan. 2001.
[12] U.S. Federal Communications Commission, *First Report and Order for UWB Technology*, U.S. Federal Communications Commission, April 2002.
[13] W. Turin, R. Jana, S. Ghassemzadeh, C. Rice, and V. Tarokh, “Autoregressive modeling of an indoor UWB channel,” in *Digest of Papers, UWBST'02 Conference,* pp. 71-74, May 2002.
[14] S. Ghassemzadeh, R. Jana, C. Rice, W. Turin, and V. Tarokh, “A statistical path loss model for in-home UWB channel,” in *Digest of Papers, UWBST'02 Conference,* pp. 59-64, May 2002.
[15] R. Bastidas-Puga, F. Ramírez-Mireles, and D. Muñoz-Rodríguez, “Performance of UWB PPM in residential multipath environments,” in *Proc. IEEE Veh. Technol. Conf. 2003 Fall,* pp. 2307-2311, Oct. 2003.