A mathematical diet model

Abstract

Emphasis on problem solving and mathematical modeling has gained considerable attention in the last few years. Connecting mathematics to other subjects and to the real world outside the classroom has received increased attention in the mathematics programs. This article describes an application of simple differential equations in the field of dietetics as it may be presented to a calculus class, or a high school mathematics club. The problem arose as a natural question about life and a mathematical model was developed to give a reasonable solution to this.

Introduction

More than at any other time in history, society is placing demands on citizens to use and interpret mathematics to make sense of information and complex situations. Problem solving and mathematical modeling are important components of learning mathematics, and these topics continue to receive significant attention in recommendations for school mathematics (NCTM 1989, 2000).

Inevitably a teacher will encounter students who see no reason to study mathematics beyond the basics. When asked why they are enrolled in a higher mathematics course, students usually mention college requirements. It then becomes the responsibility of the textbook and the instructor to generate interest in mathematics.

Teaching mathematics with practical applications in other disciplines is one of the best pedagogical tools for heightening interest in mathematics. By introducing mathematics applications in nonmathematical contexts, one can motivate students with limited initial interest in mathematics to become increasingly familiar with important mathematical tools. The teacher needs to change the emphasis of problem solving from “Here is a problem, solve it” to “Here is a situation, let’s explore it!”. The way in which a problem is presented will in large measure determine the degree to which the students’ interest will be sparked to investigate it and to make additional discoveries (Abrams, 2001).

This article describes an application of simple differential equations in the field of diet and losing weight as it may be presented to a calculus class, or a high school mathematics club. In the following pages I will present, investigate and criticize a mathematically simple model about a topic that is familiar. The model demonstrates how we use information about dietetics in order to make some predictions about the loss weighting as a function of time. This material is suitable for a class with experience of a semester on calculus, especially as a source for a group project.

Background – Formulation of the problem

Scientific evidence increasingly supports that good nutrition is essential to the health, productivity, self-sufficiency, and life quality of adults (Kerscher & Pegues, 1998). Food is material needed by all living things for growth and development and to sustain life. We get our energy from the carbohydrates stored in our food. Our bodies break down the carbohydrates into three fuels; glucose, glycogen and fats. All three of these fuels are used by our bodies to produce the heat needed to keep our bodies at a steady temperature of 37 deg C. These fuels also provide the energy needed to keep our heart beating, to breathe, walk, talk, and do everything else we expect our bodies to be able to do.

Dieting is the practice or habit of eating (and drinking) in a regulated fashion with the aim of losing (or, sometimes, gaining) weight, or in some cases to regulate the amounts of certain nutrients.

The shelves of any pharmacy are crammed with products from many nations, including “The Sunflower Diet”, “The Vitamin C diet”, “Metabolic Cell out Diet”, “The Kidney bean Diet”, etc. Yet the message is inescapable: thin, or ever thinner, is “in”. So, in most cases, diet is more a cosmetic issue than a health one. This leads to a diet mania and people often are attracted to diets and programs that promise magical, no-stress weight loss. This has led to a weight loss industry and billions of dollars are spent annually to fight obesity. Americans, for example, have spent $33 billions annually on weight-loss foods, products, and services (Golditz, 1992).

A successful weight loss diet is all about energy in vs. energy out. The heat energy is measured in units called calories (cal). One calorie of heat energy is the energy needed to raise the temperature of 1 gram of water by 1 degree C. If a person takes in fewer calories than he or she expends over a period of time, that person will burn fat and subsequently lose weight. The daily calorie requirements vary with age and gender. Likewise, the calorie requirements of physical activities vary depending upon the physical exertion required. Table (1) shows necessary calories per day for some people by age, sex, height and weight.

Age (years) and sex	**Height (cm)**	**Weight (kg)**	**Necessary calories per day**
1 year (boys + girls)	72	10	900
2 year (boys + girls)	85	12	1200
5 year (boys + girls)	102	18	1600
8 year (boys + girls)	120	24	2000
16 year boys	167	60	3800
16 year girls	157	51	2400
Not working males		65	2500
Working males			3000
Hard-working males			4200
Not working females		58	2100
Working females			2500
Hard-working females			3200

Table 1: Necessary calories
Source: (Fitrakis, 1985, p.53)

An interesting question, which can be investigated, is the relationship between the weight loss of a person who adopts a specific diet and the time in days this diet lasts.

The mathematical model

In order to create a mathematical model we have to decide what factors are relevant to the problem and what factors can be de-emphasized. The description need only be a good enough approximation to provide an adequately accurate answer and the freedom permits one to simplify the description, omitting aspects of irrelevant importance, so as to make the analysis and computation feasible (Giordano et al., 1997).

A person's weight depends both on the daily rate of energy intake, say, C calories per day, on the daily rate of energy expenditure, which is about between 35 and 45 calories per kg. per day depending on age, sex, metabolic rate, etc. For the needs of our model we shall assume that a person's daily average value of energy expenditure is 40 calories per kg per day. Therefore, a person weighting B kilograms expends $40B$ calories per day. If $C = 40B$, then weight remains constant, and weight loss or gain occurs according to whether $C < 40B$ or $C > 40B$ correspondingly.

But how fast will weight loss or gain occur? Let's we suppose that the weight B at time t in hours is $B(t)$ in kilograms. A plausible physiological assumption is that the rate of change, $\frac{dB}{dt}$ is proportional to the net deficit or excess $C - 40B$ in the number of calories per day. So we can formulate the relationship

$\frac{dB}{dt} = A(C - 40B)$ (1), where A is a constant.

In order to proceed we need to know the value of constant A. The left side of (1) has units of kg/day, and the factor $(C - 40B)$ has units calories/day. Hence, the units of A are kg/calories. Therefore, we need to know how many kg each deficit or excess takes off or puts on. The commonly used dietetic conversion factor is that 7700 calories are equivalent to one kg (Mackarness, 1988, p.47). That is, the result of consuming 7700 calories without expending any energy would be a weight gain of 1 kg. Thus, $A=\frac{1}{7700}$ (kg/calorie) and the equation (1) becomes

$\frac{dB}{dt} = \frac{1}{7700}(C - 40B)$ (2).

This is the equation modeling weight loss or gain. In order to simplify the description we further assume that the daily rate of energy intake C is constant. Under those restrictions the equation (2) is a first order separable differential equation, which can be rewritten as

$$7700\frac{dB}{dt}+40B=C$$

or $\frac{dB}{dt}+\frac{40}{7700}B=\frac{C}{7700}$

or $\frac{dB}{dt}+0.0052B(t)=\frac{C}{7700}$ (3).

Multiplying both sides by $e^{0.0052t}$, (3) gives the equation

$$e^{0.0052t}\frac{dB}{dt}+0.0052e^{0.0052t}B(t)=\frac{C}{7700}e^{0.0052t},$$

$$\frac{d}{dt}\left[e^{0.0052t}B(t)\right]=\frac{C}{7700}e^{0.0052t},$$

$$\frac{d}{dt}\left[e^{0.0052t}B(t)\right]=\frac{d}{dt}\left[\frac{Ce^{0.0052t}}{7700\cdot 0.0052}\right],$$

$e^{0.0052t}B(t)=\frac{C}{40}e^{0.0052t}+K$ (4), where K is a constant.

For $t = 0$ (4) yields $K = B(0) - \frac{C}{40}$, where $B(0)\equiv B_0$ is the initial weight. After that we have $B(t)=\frac{C}{40}+\left(B_0-\frac{C}{40}\right)e^{-0.0052t}$ (5), the weight function, where t is in days.

Interpreting and validating the results.

Once the problem is solved on the basis of our original assumptions our resulting model solution must be analyzed and interpreted with respect to the problem. The following questions are useful for this aim:

- Is the information produced reasonable?
- Are the assumptions made while developing the model reasonable?
- How do the results compare with the real data?

There are certainly other plausible assumptions about the rate of change $\frac{dB}{dt}$ or the daily rate of energy intake C, but the ones chosen above seem to be the simplest that can still reflect some of the complexities.

There are as many different recommendations for supposedly successful diets as there are people trying to lose weight. What works for one person will not necessarily work for another due to metabolic differences and lifestyle factors. Weight control can be a highly reactive subject. Millions of people have tried countless times to succeed at the battle of the bulge only to watch the kilos creep back on time and time again.

Let's now take a look at what information our dietetic model can provide us with and how useful this information can be to those wishing to follow a specific diet according to this. Let's suppose that a plump lady, 35 years of age, is 1.65 m high and weighs 85 kilos. She decides therefore, to become more elegant so as to feel more attractive to the male sex (she is interested in getting married !!). Thus, she starts a diet of 2500 calories per day, which, according to table 1, are considered sufficient for the daily needs of the organism, even for an averagely working person. Hence, the relationship (5) which express the weight as a function of time in days becomes:

$$B(t) = \frac{2500}{40} + \left(85 - \frac{2500}{40}\right)e^{-0.0052t} \quad \text{or} \quad B(t) = 62.5 + 22.5e^{-0.0052t} \qquad (6).$$

A graphic calculator or graph-plotter allows the graph to be drawn directly from the formula (6). Figure (2) shows the graph of this function and provides a visual interpretation of the results.

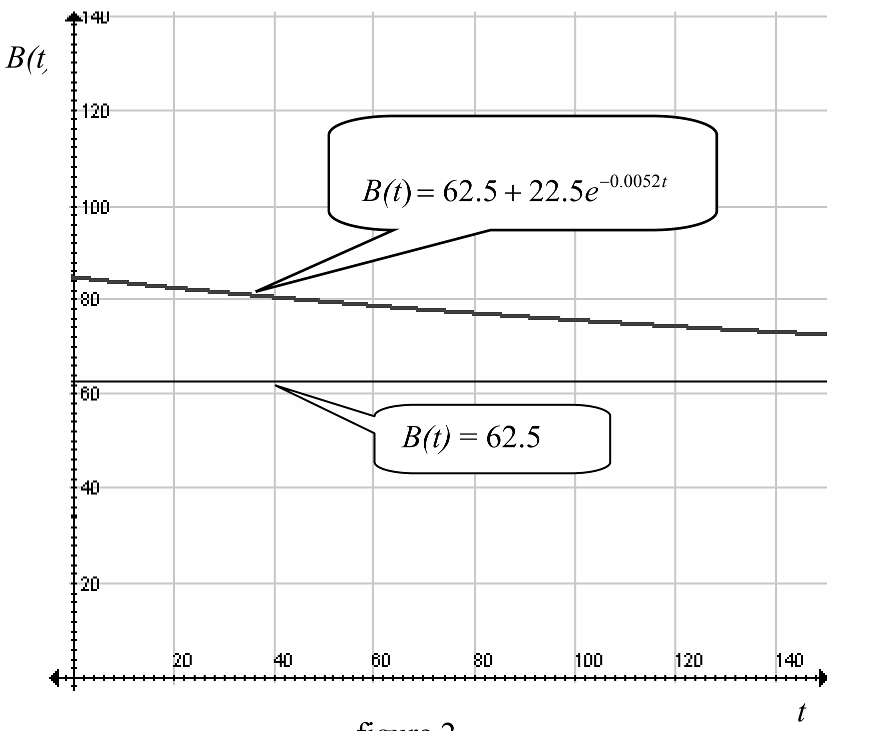

figure 2

Weight loss with constant caloric intake $C = 2500$, for $B_0 = 85$ kg

- Since $\lim_{t \to +\infty} e^{-0.0052t} = 0$, then $\lim_{t \to +\infty} B(t) = 62.5$. This result shows us how long it takes to even get close to the asymptotic weight of 62.5 kg.

- In one year (365 days) our lady will lose about 19 kg, provided that she bears that diet of course.

- From that point on, the kilos are lost with great difficulty because many days are needed. For instance, to lose 3 more kilos, that is 22 in total, 732 days are required, almost 2 years. From that point onwards however, in order to lose half a kilo more and reach 62.5 kg, not even her whole life is enough to her!!

- If our lady wants to go down to less than 62.5 kg and reach, let's say, 58, which is the ideal weight for her height, she must reduce the number of calories she takes every day, which however puts her health to risk. If for example, the number of calories is reduced to 2000 the day, we have:
  $$B(t) = \frac{2000}{40} + \left(85 - \frac{2000}{40}\right) e^{-0.0052t} = 50 + 35e^{-0.0052t}.$$
  If we want the weight to fall to 58kg we have, $58 = 50 + 35e^{-0.0052t}$ or $e^{-0.0052t} = 0.23$ and taking logarithms it yields $t \cong 282$ days or about 40 weeks. However, according to table 1, with 2000 calories, the needs of an 8-year old child that weighs 24 kg are barely satisfied. How is it possible for a working person that weighs at least 60 kg to be fed sufficiently?

- From the graphical representation we can see how long a time period is required for the weight to reach the asymptotical price of 62.5 kg. This perhaps explains why so many of those who start a strict and ambitious diet, usually abandon it disappointed.

These results make the model appear in some way reasonable, as it seems to fit well with our perception of reality. We interpret the model in only a rough estimation, according to our original assumptions and acceptances.

Concluding remarks

Models can be used to identify trends and make general forecasts. Models are not reality; they are extreme simplifications of reality. In truth, we rarely, if ever, obtain a mathematical model, which perfectly captures the data we are studying.

The ideas treated here could be used in one of at least two ways. The problem of modeling weight gain or loss could be put to a calculus class as an extended laboratory exercise or project. The subject matter of the article could serve also as the text of a talk to a mathematics class or club.

Engaging students in a model building process like this is likely to foster a diversity, creativity, and richness of responses far beyond that encountered in most mathematics or science classrooms, especially those where a premium is placed on following a correct procedure. Introducing such real life problems into mathematics courses can generate the student's interest in mathematics as a valuable tool for them.

REFERENCES

Abrams, J.P. (2001) Teaching Mathematical Modeling and Skills of Representation, *In The Roles of Representation in School Mathematics*, 2001 Yearbook, NCTM,(Eds. Cuoco,A.A. and Curcio, F.R.), pp. 269 – 282.

Fitrakis. P. (1985) *Modern dietitic*. Fitrakis publishing, Athens.

Giordano,F., Weir, M., and Fox, W. (1997) *A first Course in mathematical Modeling*, 2nd ed., brooks/Cole Publishing, Pacific Grove, CA.

Golditz, G. (1992) Economic costs of obesity. *Am. J. Clin. Nutr*., 55: 503 – 507.

Keschner, H. and Pegues, J. M. (1998) Productive aging: a quality of life agenda. *J. Am.. Diet Assoc.*, 98, 1445 – 1448.

Mackarness, R. (1988) *How to weaken eating*. Fitrakis publishing, Athens.

National Council of Teachers of Mathematics. (1988) *Curriculum and Evaluation Standards for School Mathematics*, Reston, VA: The Council.

National Council of Teachers of Mathematics. (2000) *Principles and Standards for School Mathematics*, Reston, VA:, The Council.