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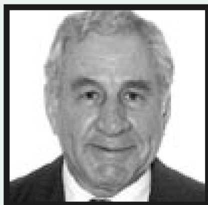

Joe Portney

Portney's Corner

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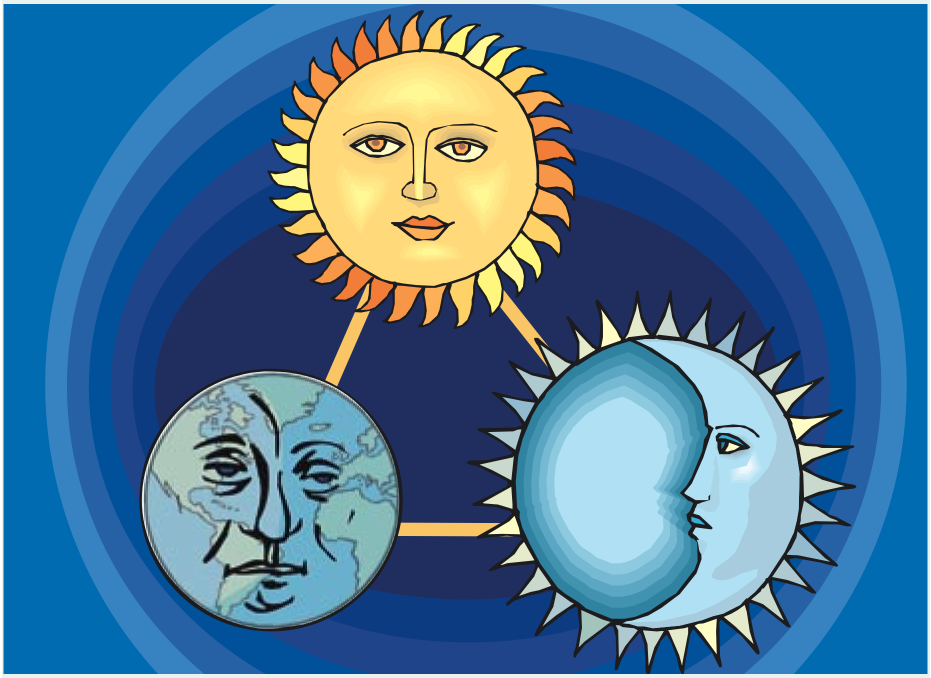

The Lunar Distance Method

Before GPS and chronometers, we depended on lunar distances. Longitude can be determined using the Lunar Distance Method without using a chronometer. The method became practical in the late 1700s and was used until the early 1900s. By the late 1700s, the mathematics of spherical trigonometry was well established, and the ephemerides of the sun and moon could be calculated to sufficient accuracy for practical longitude determination.

The moon loses a full circle to the sun in about 29.5 days. The angle between the sun and moon acts like the hands of a giant clock with the angle changing roughly 30.5 seconds of arc in a minute of time. If the positions of the sun and moon could be predicted well enough in advance, the angle between the sun and the moon (the lunar distance) could be tabulated as a function of Greenwich Mean Time (GMT). The sun and moon could serve as an astronomical clock.

Portney *continued on page 14*

CALENDAR

SEPTEMBER 2004

21–24: ION GNSS 2004, Long Beach California
Contact: The ION
Tel: +1 703-383-9688
Fax: +1 703-383-9689
Web: www.ion.org

OCTOBER 2004

05-06: International Symposium on Precision Approach and Automatic Landing, Munich, Germany
Contact: German Institute of Navigation
Tel: +49-228-20197.0
Fax: +49-228-20197.19
E-mail: dgon.bonn@t-online.de

25-27: ILA Convention and Technical Symposium, Takanawa Prince Hotel, Tokyo, Japan
Contact: ILA
Tel: +1-805-967-8649
Fax: +1-805-967-8471
E-mail: ila@loran.org

JANUARY 2005

24–26: ION National Technical Meeting, San Diego, California
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MAY 2005

23-25: The 12th St. Petersburg International Conference on Integrated Navigation Systems, St. Petersburg, Russia
Contact: Dr. George T. Schmidt
Tel: +1 617-258-3841
e-mail: ED@Draper.com
Web: www.elektropribor.spb.ru

JUNE 2005

27-29: The ION 61st. Annual Meeting; Royal Sonesta Hotel, Cambridge, Massachusetts
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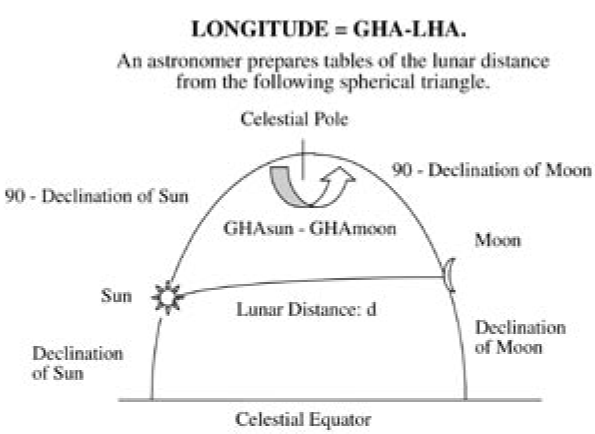
Portney's Corner *continued from page 5*

In practice, the lunar distance method involved taking three sextant observations: (1) the altitude of the moon above the horizon, (2) the altitude of the sun above the horizon, and (3) the angular distance between the sun and the moon. Spherical trigonometric calculations were performed to compensate the measured lunar distance for refraction error and parallax error. Once the true lunar distance was determined, GMT could be determined by interpolating in tables of lunar distance versus GMT. Once GMT was known, other tables could be read to determine the declination and Greenwich Hour Angle (GHA) of the sun and moon. Given that the latitude was known, another spherical triangle could be solved for the Local Hour Angle (LHA) of the sun. The longitude followed from the relationship shown in Figure 1.

Figure 1.



The declination and GHA of the sun and moon would be known from the almanac. Two sides and the included angle are known in the spherical triangle. One solves for the third side d using the law of cosines for spherical triangles. After some simplifications, the result is:

$$\cos(d) = \sin(DECsun)\sin(DECmoon) + \cos(DECsun)\cos(DECmoon)\cos(GHAsun - GHAmoon)$$

Tables of lunar distances were published up until the early 1900s (Figure 4). Today, a user would have to prepare the tables himself from the declinations and GHA of the sun and moon using spherical trigonometry.

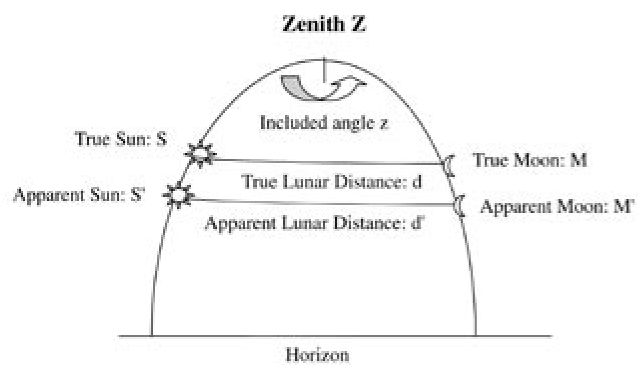
This is not difficult with modern computers, but is an extremely laborious task if done by hand.

The observed lunar distance is calculated from the results of three sextant observations, ideally made simultaneously:

1. Altitude of the lower limb of the sun
2. Altitude of the lower limb of the moon
3. Apparent distance d' between the bright limbs of the sun and the moon

A spherical triangle is solved mainly to remove the effects of parallax and refraction error from the apparent lunar distance measurement. (See Figure 2.)

Figure 2.



Let $h' = 90 - ZM'$: Apparent Altitude of Moon's Center Corrected for Dip
Let $H' = 90 - ZS'$: Apparent Altitude of Sun's Center Corrected for Dip
Let d' = Apparent distance between Moon and Sun's Center

Corrections for refraction and parallax occur in altitude. After correcting the observed altitudes for these effects, we get:

$h = 90 - ZM$: Moon's true altitude
$H = 90 - ZS$: Sun's true altitude

We then need to calculate the side SM, the true lunar distance d. The key to this calculation is the fact that the parallax and refraction corrections are in altitude only and do not alter the included angle z in the triangles. The angle z is common to the spherical triangles ZS'M' and ZSM.

One procedure which is practical with modern computers is to use the law of cosines for spherical triangles to solve the triangle ZS'M' for cos(z):

$$\cos(d') = \sin(h')\sin(H') + \cos(h')\cos(H')\cos(z)$$

The rest of the terms in the above equation are known from sextant observations corrected for dip and apparent center effects. Then the triangle ZSM can be solved for the true lunar distance:

$$\cos(d) = \sin(h)\sin(H) + \cos(h)\cos(H)\cos(z)$$

All the terms on the right hand side of this equation are known. H and h are known from measured altitudes corrected for parallax and refraction. Cos(z) was obtained from the triangle ZS'M'.

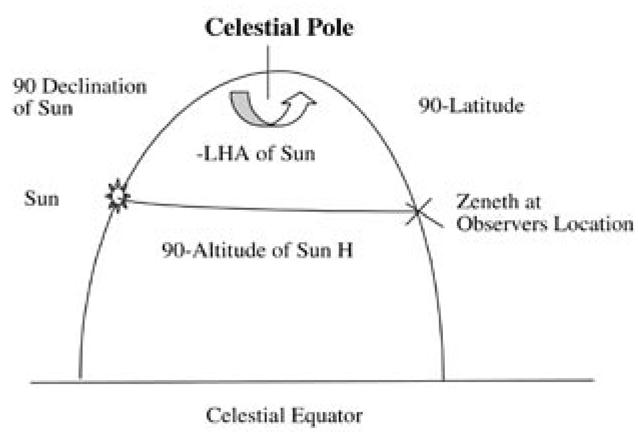
In olden days, more elaborate procedures were used which were easier to use with tables and hand calculations.

Once the true lunar distance d is obtained, one can interpolate in tables of lunar distance versus GMT to find the Greenwich Mean Time. This time could be used to calibrate a chronometer. Alternatively, if latitude is known, spherical trigonometry can be used to find the Local Hour Angle (LHA) of the sun.

If GMT is known, one can look in tables to find the declination and Greenwich Hour Angles (GHA) of the sun and moon. (See Figure 4.)

This triangle can be solved for LHA of the Sun once the GMT is known. The Altitude of the Sun H is known from corrected sextant observations.

Figure 3.



$$\sin(H) = \sin(DECsun)\sin(latitude) + \cos(DECsun)\cos(latitude)\cos(LHA)$$

Once the LHA of the sun is determined, longitude follows from the relationship:

$$Longitude = GHAsun - LHAsun$$

Similar calculations could be carried out for the moon as a check.

Accurate chronometers were developed by John Harrison around 1759 and by French Clockmaker Berthoud a few years later. Longitude could be determined using a chronometer with much less calculation than the lunar distance method. The Lunar distance method did see some use in the 1800s as an alternative to chronometers, and a way to recover time if a chronometer were not available.

References

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Stanley W. Gery, "The Direct Fix of Latitude and Longitude from Two Observed Altitudes," *Navigation, Journal of the Institute of Navigation,* Vol. 44, No. 1, Spring 1997.

—*We are indebted to David F. Hartman the author of this paper entitled* "The Lunar Distance Method." *Mr. Hartman is employed at ATK Missile Systems Company in Woodland Hills, Ca. A copy of an extract of the first systematic tabulation of lunar distances is shown in Figure 4. It was published by Nevil Maskelyne, Astronomer Royale of the Greenwich Observatory.*

Figure 4.

[36] MARCH 1767.

Distances of ☽'s Center from ☉, and from Stars west of her.

Days.	Stars Names.	12 Hours. ° ′ ″	15 Hours. ° ′ ″	18 Hours. ° ′ ″	21 Hours. ° ′ ″
3	The Sun.	47. 35. 32	49. 14. 7	50. 52. 15	52. 29. 58
4		60. 31. 52	62. 6. 53	63. 41. 28	65. 15. 37
5		72. 59. 57	74. 31. 33	76. 2. 45	77. 33. 33
6		85. 1. 42	86. 30. 12	87. 58. 22	89. 26. 11
7		96. 40. 30	98. 6. 26	99. 32. 5	100. 57. 27
8		108. 0. 35	109. 24. 27	110. 48. 7	112. 11. 34
9		119. 6. 21			
6	Arietis.	36. 56. 52	38. 32. 5	40. 7. 2	41. 41. 42
7		49. 30. 50	51. 3. 51	52. 36. 36	54. 9. 5
8	Aldebaran.	30. 50. 33	32. 16. 45	33. 43. 9	35. 9. 45
9		42. 24. 35	43. 51. 40	45. 18. 45	46. 45. 51
10		54. 1. 29	55. 28. 35	56. 55. 42	58. 22. 48
11		65. 38. 23	67. 5. 29	68. 32. 35	69. 59. 42
12	Pollux.	34. 42. 35	36. 10. 18	37. 38. 5	39. 5. 57
13		46. 26. 22	47. 54. 39	49. 23. 0	50. 51. 26
14	Regulus.	21. 13. 28	22. 42. 27	24. 11. 34	25. 40. 48
15		33. 8. 32	34. 38. 25	36. 8. 24	37. 38. 30
16		45. 10. 46	46. 41. 36	48. 12. 32	49. 43. 36
17		57. 20. 54	58. 52. 45	60. 24. 45	61. 56. 54
18	Spica.	15. 48. 39	17. 20. 33	18. 52. 46	20. 25. 17
19		28. 12. 30	29. 46. 44	31. 21. 15	32. 65. 1
20		40. 53. 41	42. 30. 0	44. 6. 34	45. 43. 27
21		53. 51. 55	55. 30. 27	57. 9. 17	58. 48. 25
22		67. 8. 48	68. 49. 50	70. 31. 11	72. 12. 52
23	Antares.	34. 55. 40	36. 39. 31	38. 23. 42	40. 8. 14
24		48. 56. 5	50. 42. 38	52. 29. 31	54. 16. 45
25		63. 17. 49	65. 6. 58	66. 56. 24	68. 46. 7
26	Capricorni.	23. 43. 43	25. 32. 9	27. 21. 11	29. 10. 44
27	Aquilae.	46. 57. 42	48. 20. 33	49. 44. 53	51. 10. 35
28		58. 36. 8	60. 7. 55	61, 40. 21	63. 13. 19