We replace ε in Eq. A.31 to arrive at this equation.

$$p = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{3}{2(M-1)} \cdot \frac{\varepsilon_{AV}\left(\sum_{k=1}^{L}\alpha_k w_k\right)^2}{N_0\sum_{k=1}^{L}w_k^2}}\right) \quad \text{(A.39)}$$

We define γ as follows.

$$\gamma = \frac{\varepsilon_{AV}\left(\sum_{k=1}^{L}\alpha_k w_k\right)^2}{N_0\sum_{k=1}^{L}w_k^2} \quad \text{(A.40)}$$

Replacing γ into the above equation for p results in Eq. 2.36.

A.2.3 Derivation of P_e for Rectangular Constellations

We shall assume a rectangular constellation of dimension $\sqrt{\frac{M}{2}}$ by $\sqrt{2M}$. The probability of error for this constellation provides an upper bound to the probability of error of the optimal constellation. With the same approach as before, we have 4 signals with two neighbors, $(3\sqrt{2M}-8)$ signals with three neighbors, and $(M-3\sqrt{2M}+4)$ signals with four neighbors. Therefore,

$$P_e(\gamma) < 1 - \frac{1}{M}\left[4p_2 + (3\sqrt{2M}-8)p_3 + (M-3\sqrt{2M}+4)p_4\right] \quad \text{(A.41)}$$

Replacing the expressions for p_2, p_3, and p_4 from Eqs. A.8 - A.13 into the above equation,

$$P_e(\gamma) < 2\left(2-\frac{3}{\sqrt{2M}}\right)p - \left(2-\frac{3}{\sqrt{2M}}\right)^2 p^2 + \frac{1}{2}p^2 \quad \text{(A.42)}$$

The p^2 terms will be small with respect to p. Therefore a tight bound is,

$$P_e(\gamma) < 2\left(2-\frac{3}{\sqrt{2M}}\right)p \quad \text{(A.43)}$$

The equation for p is the same as in Eq. A.31. But ε will be different since we have a rectangular constellation instead of a square. To determine ε, we shall first define the signal amplitudes in terms of the inphase and quadrature components.

$$A_{mc} = 2m - 1 - \sqrt{2M} \quad m = 1, 2, \ldots \sqrt{2M} \tag{A.44}$$

$$A_{ms} = 2m - 1 - \sqrt{\frac{M}{2}} \quad m = 1, 2, \ldots \sqrt{\frac{M}{2}} \tag{A.45}$$

Therefore the average signal amplitude is equal to,

$$E[A_m^2] = \frac{2M-1}{3} + \frac{\frac{M}{2}-1}{3} \tag{A.46}$$

$$= \frac{\frac{5}{2}M - 2}{3} \tag{A.47}$$

$$= \frac{2\left(\frac{5}{4}M - 1\right)}{3} \tag{A.48}$$

And finally the average energy is expressed as

$$\varepsilon_{AV} = E[A_m^2]\varepsilon \quad [51] \tag{A.49}$$

$$= \frac{2}{3}\left(\frac{5}{4}M - 1\right)\varepsilon \tag{A.50}$$

From this we have an expression for ε.

$$\varepsilon = \frac{3\varepsilon_{AV}}{2\left(\frac{5}{4}M - 1\right)} \tag{A.51}$$

By replacing ε in Eq. A.31 with this result,

$$p = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{3}{2\left(\frac{5}{4}M-1\right)} \cdot \frac{\varepsilon\left(\sum_{k=1}^{L}\alpha_k w_k\right)^2}{N_0 \sum_{k=1}^{L} w_k^2}}\right) \tag{A.52}$$

By setting,

$$\gamma = \frac{\varepsilon_{AV}\left(\sum_{k=1}^{L}\alpha_k w_k\right)^2}{N_0\sum_{k=1}^{L} w_k^2} \tag{A.53}$$

we have the final equation for p.

$$p = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\cdot\sqrt{\frac{3}{2\left(\frac{5}{4}M-1\right)}}\right) \tag{A.54}$$

Substituting the above equation for p into Eq. A.43,

$$P_e(\gamma) < \left(2-\frac{3}{\sqrt{2M}}\right)\mathrm{erfc}\left(\sqrt{\gamma}\cdot\sqrt{\frac{3}{2\left(\frac{5}{4}M-1\right)}}\right) \tag{A.55}$$

$$= 2\left(1-\frac{\frac{3}{2\sqrt{2}}}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\gamma}\cdot\sqrt{\frac{3}{2\left(\frac{5}{4}M-1\right)}}\right) \tag{A.56}$$

If we replace the $\left(1-\frac{1}{\sqrt{M}}\right)$ terms with $\left(1-\frac{\frac{3}{2\sqrt{2}}}{\sqrt{M}}\right)$ and the $(M-1)$ terms with $\left(\frac{5}{4}M-1\right)$ in Eq. 2.39, the result is P_e.

$$P_e < \frac{2(1+(L-1)\rho)^{L-1}\left(1-\frac{\frac{3}{2\sqrt{2}}}{\sqrt{M}}\right)(1-u_1)}{(L\rho)^{L-1}} - \left(1-\frac{\frac{3}{2\sqrt{2}}}{\sqrt{M}}\right)(1-\rho)\sum_{i=0}^{L-2}\frac{(1+(L-1)\rho)^{L-2-i}(1-u_2)^{i+1}}{(L\rho)^{L-1-i}2^{i-1}} * \sum_{k=0}^{i}\begin{pmatrix} i+k \\ k \end{pmatrix}\left(\frac{1+u_2}{2}\right)^k \tag{A.57}$$

where

$$u_1 = \sqrt{\frac{3\overline{\gamma_c}(1+(L-1)\rho)}{2\left(\frac{5}{4}M-1\right)+3\overline{\gamma_c}(1+(L-1)\rho)}} \tag{A.58}$$

$$u_2 = \sqrt{\frac{3\overline{\gamma_c}(1-\rho)}{2\left(\frac{5}{4}M-1\right)+3\overline{\gamma_c}(1-\rho)}} \tag{A.59}$$

A.3 Appendix for Non-coherent FSK

A.3.1 Derivation of $[P(U_2 < U_1 | U_1 = u_1)]^{M-1}$

We substitute $p(U_2)$ from Eq. 2.52 into Eq. 2.56.

$$P(U_2 < U_1 | U_1 = u_1) = \int_0^{u_1} \frac{1}{(2\sigma_2^2)^L (L-1)!} u_2^{L-1} e^{-u_2/2\sigma_2^2} \, du_2 \qquad \text{(A.60)}$$

From [9],

$$\int x^m e^{ax} \, dx = e^{ax} \sum_{r=0}^{m} (-1)^r \frac{m! \, x^{m-r}}{(m-r)! \, a^{r+1}} \qquad \text{(A.61)}$$

Using the above equation, we can solve the integral in Eq. A.60.

$$P(U_2 < U_1 | U_1 = u_1) = 1 - \sum_{r=0}^{L-1} \frac{u_1^{L-1-r} e^{-u_1/2\sigma_2^2}}{(L-1-r)! \, (2\sigma_2^2)^{L-1-r}} = 1 - \sum_{j=0}^{L-1} y_j \qquad \text{(A.62)}$$

Raising Eq. A.62 to the $M-1$ power, and using [22],

$$(a+x)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} x^k a^{n-k} \qquad \text{(A.63)}$$

we obtain,

$$\left(1 - \sum_{j=0}^{L-1} y_j\right)^{M-1} = \sum_{r=0}^{M-1} (-1)^r \begin{pmatrix} M-1 \\ r \end{pmatrix} \left[\sum_{j=0}^{L-1} y_j\right]^r \qquad \text{(A.64)}$$

In the following equations we remove the first y_j from the summation and reapply Eq. A.63.

$$\left(1 - \sum_{j=0}^{L-1} y_j\right)^{M-1} = \sum_{r=0}^{M-1} (-1)^r \begin{pmatrix} M-1 \\ r \end{pmatrix} \left[y_0 + \sum_{j=1}^{L-1} y_j\right]^r$$

$$= \sum_{r_1=0}^{M-1} \sum_{r_2=0}^{r_1} \cdots \sum_{r_L=0}^{r_{L-1}} (-1)^{r_1} \begin{pmatrix} M-1 \\ r_1 \end{pmatrix} \cdot$$

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \cdots \begin{pmatrix} r_{L-1} \\ r_L \end{pmatrix} y_0^{r_1-r_2} y_1^{r_2-r_3} \cdots y_{L-2}^{r_{L-1}-r_L} y_{L-1}^{r_L} \quad \text{(A.65)}$$

If we combine the results from Eq. A.62 and Eq. A.65 we get the following.

$$\begin{aligned} &[P(U_2 < U_1 | U_1 = u_1)]^{M-1} = \\ &\sum_{r_1=0}^{M-1} \sum_{r_2=0}^{r_1} \cdots \sum_{r_L=0}^{r_{L-1}} (-1)^{r_1} \begin{pmatrix} M-1 \\ r_1 \end{pmatrix} \prod_{j=1}^{L-1} \begin{pmatrix} r_j \\ r_{j+1} \end{pmatrix} \cdot \\ &\prod_{i=0}^{L-2} \left[\frac{1}{(L-1-i)!\,(2\sigma_2^2)^{L-1-i}} \right]^{r_{i+1}-r_{i+2}} u_1^{\sum_{k=1}^{L-1}(L-k)(r_k-r_{k+1})} e^{\frac{-u_1 r_1}{2\sigma_2^2}} \end{aligned} \quad \text{(A.66)}$$

A.3.2 Derivation of P_e

The probability of error in terms of the probability of a correct decision is $P_e = 1 - P_c$. We have the equation for P_c in Eq. 2.55. Substitute Eq. A.66 and Eq. 2.47 into this equation to obtain

$$\begin{aligned} P_c \;=\; &\sum_{r_1=0}^{M-1} \sum_{r_2=0}^{r_1} \cdots \sum_{r_L=0}^{r_{L-1}} (-1)^{r_1} \begin{pmatrix} M-1 \\ r_1 \end{pmatrix} \prod_{j=1}^{L-1} \begin{pmatrix} r_j \\ r_{j+1} \end{pmatrix} \cdot \\ &\prod_{i=0}^{L-2} \left[(L-1-i)! \left(2\sigma_2^2\right)^{L-1-i} \right]^{r_{i+1}-r_{i+2}} \cdot \\ &\left\{ \frac{(\sigma_1^2 + (L-1)\mu)^{L-2}}{2(L\mu)^{L-1}} \int_0^\infty u_1^{\sum_{k=1}^{L-1}(L-k)(r_k-r_{k+1})} \cdot \right. \\ &\exp\left[-u_1 \left(\frac{1}{2(\sigma_1^2 + (L-1)\mu)} + \frac{r_1}{2\sigma_2^2} \right) \right] du_1 - \end{aligned}$$

$$\sum_{i=0}^{L-2} \frac{(\sigma_1^2 + (L-1)\mu)^{L-2-i}}{2^{i+1} \cdot (L\mu)^{L-1-i} \cdot (\sigma_1^2 - \mu)^i \cdot i!} \int_0^\infty u_1^{\sum_{k=1}^{L-1}(L-k)(r_k - r_{k+1}) + i} \cdot$$
$$\exp\left[-u_1 \left(\frac{1}{2(\sigma_1^2 - \mu)} + \frac{r_1}{2\sigma_2^2}\right)\right] du_1 \Bigg\} \qquad (A.67)$$

The following formula from [22] may be employed to solve the integrals:

$$\int_0^\infty x^{\nu-1} e^{-\mu x}\, dx \quad = \quad \frac{\Gamma(\nu)}{\mu^\nu} \qquad (A.68)$$
$$\text{where} \quad \Re\{\mu\} > 0 \quad \text{and} \quad \{Re\{\nu\} > 0$$

The resulting expression for P_c is more usefully expressed in terms of $\overline{\gamma_c}$ instead of σ_1^2, σ_2^2, and μ, where

$$\overline{\gamma_c} = \frac{\varepsilon E[\alpha^2]}{N_0}$$
$$\sigma_1^2 = \sigma_2^2(1 + \overline{\gamma_c})$$
$$\mu = \rho \overline{\gamma_c} \sigma_2^2$$

We make the substitutions to obtain Eq. 2.57 for P_e.

Appendix B

Indoor Radio Channel

B.1 Ray Tracing

B.1.1 Projections

As described in Section 3.2, the elapsed time based on the x-axis for up to four bounces is,

$$\text{0 - reflections} \quad : \quad \left| \frac{x_r - x_t}{c \cos\theta_t \cos\phi_t} \right| \tag{B.1}$$

$$\text{1 - reflection} \quad : \quad \left| \frac{x_t + x_r}{c \cos\theta_t \cos\phi_t} \right| \tag{B.2}$$

$$: \quad \left| \frac{(W - x_t) + (W - x_r)}{c \cos\theta_t \cos\phi_t} \right| \tag{B.3}$$

$$\text{2 - reflections} \quad : \quad \left| \frac{x_t + W + (W - x_r)}{c \cos\theta_t \cos\phi_t} \right| \tag{B.4}$$

$$: \quad \left| \frac{(W - x_t) + W + x_r}{c \cos\theta_t \cos\phi_t} \right| \tag{B.5}$$

$$3\text{ - reflections} \quad : \quad \left|\frac{x_t + 2W + x_r}{c\cos\theta_t\cos\phi_t}\right| \tag{B.6}$$

$$: \quad \left|\frac{(W - x_t) + 2W + (W - x_r)}{c\cos\theta_t\cos\phi_t}\right| \tag{B.7}$$

$$4\text{ - reflections} \quad : \quad \left|\frac{x_t + 3W + (W - x_r)}{c\cos\theta_t\cos\phi_t}\right| \tag{B.8}$$

$$: \quad \left|\frac{(W - x_t) + 3W + x_r}{c\cos\theta_t\cos\phi_t}\right| \tag{B.9}$$

where θ_t is the aspect angle and ϕ_t is the elevation angle. The projections are illustrated in Figure B.1. For the times for the y-axis and z-axis, replace the denominator by $c\sin\theta_t\cos\phi_t$ and $c\sin\phi_t$, respectively, and we replace the x-coordinates with the appropriate y and z coordinates.

B.1.2 Attenuation by Inner Walls

A ray passes through the partitions with some attenuation, but we neglect any reflections off the partitions. This greatly simplifies our model. The rationale is that the likelihood of these reflections reaching the receiver is minute. In addition the power level of the reflected ray in the event that it reaches the receiver will be extremely small due to the minimal reflection of the partition and the large path loss.

Since the path of the ray and the locations of the partitions are known, the intersection between the ray and any of the partitions can be found. The number of intersections is counted and the power level of the ray is scaled accordingly. The intersection between the ray and a partition is found as follows.

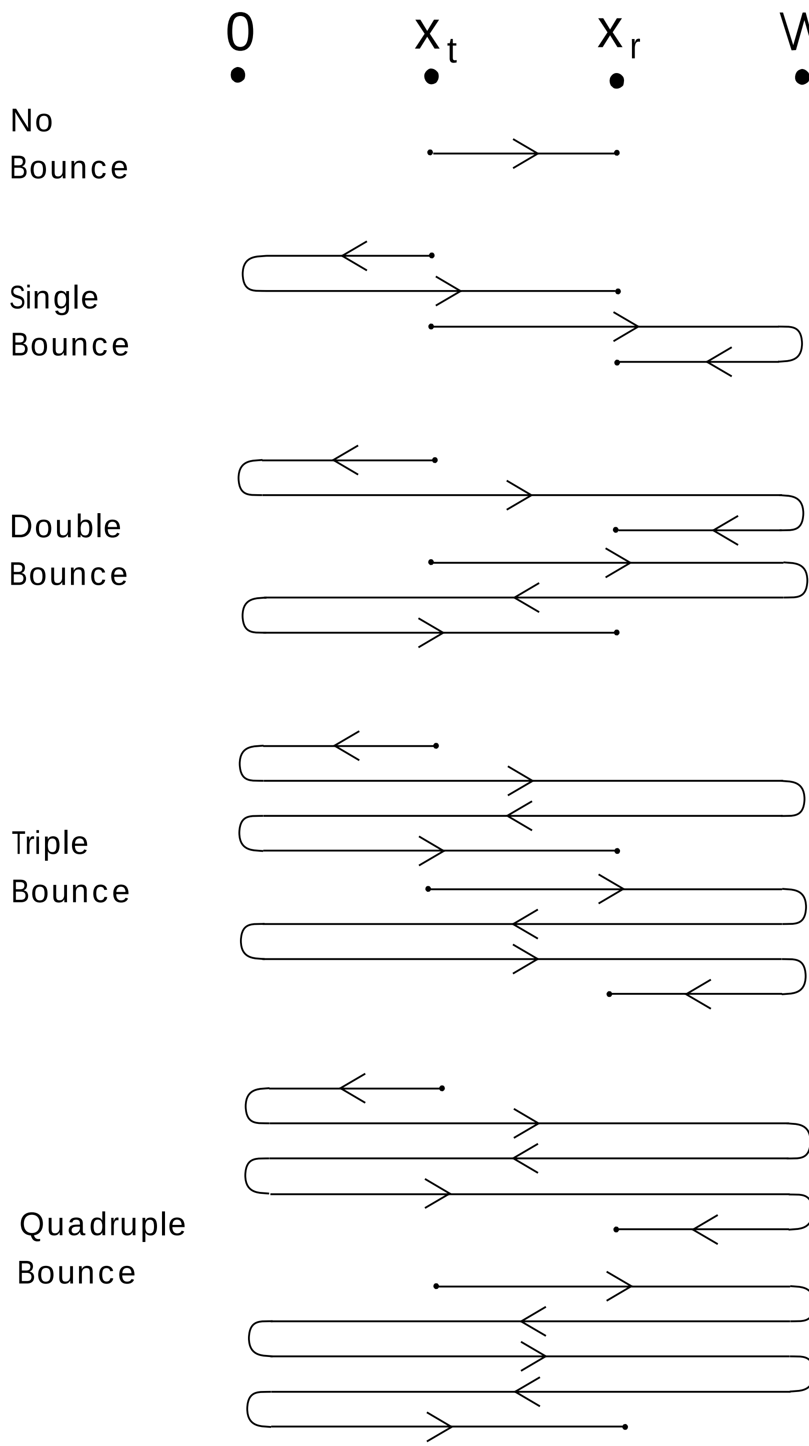
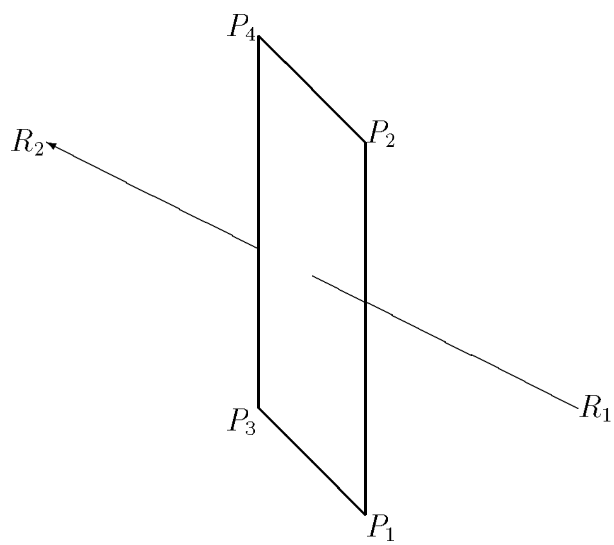


Figure B.1: Projections of the path onto the x-axis for all possible bounce combinations.

We begin by assuming that the partitions are parallel to either the walls or the floor. Select two consecutive points along the path of the ray. These include the transmitter, reflection, and receiver points. Label the points as R_1 and R_2. The four corners of the partition are labeled P_1, P_2, P_3, and P_4. See the figure below for an illustration of the problem.



Any point along the R_1-R_2 line is given by the coordinates (x, y, z) and defined by the following parametric equations.

$$x = x_{R_1} + (x_{R_2} - x_{R_1})\,t \tag{B.10}$$

$$y = y_{R_1} + (y_{R_2} - y_{R_1})\,t \tag{B.11}$$

$$z = z_{R_1} + (z_{R_2} - z_{R_1})\,t \tag{B.12}$$

If the partition is parallel to the left and right walls, then the x-coordinates of the partition points are all equal and value of x at the intersection of the ray and the partition will be equal to this value. If the partition is parallel to the top and bottom walls then y is known, and if the partition is parallel to the floor and ceiling (i.e. a false ceiling) then z is known.

One of the coordinates of the intersection point between the partition and the ray is known. This allows for solving for the parametric value t. Then the two other intersection coordinates can be found. We then check that the intersection coordinates lie between the points R_1 and R_2. If this is the case then the ray passes through the partition and the power level is attenuated by the absorption factor of the partition.

B.2 Equalizers

B.2.1 Diversity Combiner Followed by a Linear Equalizer

We solve for the equalizer tap weights that minimize the MSE. The MSE is given by the equations below.

$$\begin{aligned}
\text{MSE} &= E\left[\left|\hat{x}_n^{sig} - x_n^{sig}\right|^2\right] \qquad \text{(B.13)} \\
&= \sigma_x^2 \underbrace{\sum_{j=-N}^{N}\sum_{i=0}^{M}\sum_{q=-N}^{N}\sum_{p=0}^{M}}_{i+j=p+q} c_j^* c_q g_i^{sig} g_p^{sig*} - \sigma_x^2 \underbrace{\sum_{j=-N}^{N}\sum_{i=0}^{M}}_{i+j=0} c_j^* g_i^{sig} - \\
&\quad \sigma_x^2 \underbrace{\sum_{j=-N}^{N}\sum_{i=0}^{M}}_{i+j=0} c_j g_i^{sig*} + \sigma_x^2 +
\end{aligned}$$

$$\sigma_x^2 \underbrace{\sum_{j=-N}^{N} \sum_{i=0}^{M} \sum_{q=-N}^{N} \sum_{p=0}^{M}}_{i+j=p+q} c_j^* c_q \sum_{l=1}^{I} g_i^{int,l} g_p^{int,l*} + \sigma_{\hat{z}}^2 \sum_{j=-N}^{N} c_j c_j^* \tag{B.14}$$

As in Section 3.3.1, we rewrite the equation for the MSE in vector form. The $(2N+1)$ by $(2N+1)$ matrix $\mathbf{\Phi}$ is defined as follows.

$$\begin{aligned} \mathbf{\Phi} &= [\phi(j+N+1, q+N+1)] & \text{(B.15)} \\ &= \sigma_x^2 \underbrace{\sum_{i=0}^{M} \sum_{p=0}^{M}}_{i+j=p+q} \left\{ g_i^{sig} g_p^{sig*} + \sum_{l=1}^{I} g_i^{int,l} g_p^{int,l*} \right\} + \begin{cases} \sigma_{\hat{z}}^2 & j = q \\ 0 & \text{otherwise} \end{cases} & \text{(B.16)} \\ \text{where} \quad & j, q = -N, -N+1, \ldots N-1, N \end{aligned}$$

In addition $\mathbf{c}$ and $\mathbf{s}$ are defined as,

$$\mathbf{c} = [c_j] \tag{B.17}$$

If N is less than or equal to M then $\mathbf{s}$ has the following form.

$$\mathbf{s}^T = \sigma_x^2 [\underbrace{g_N^{sig} \; g_{N-1}^{sig} \; \cdots \; g_1^{sig} \; g_0^{sig}}_{N+1} \; \underbrace{0 \; \cdots \; 0}_{N}] \tag{B.18}$$

Otherwise if N is greater than M then $\mathbf{s}$ has the following form.

$$\mathbf{s}^T = \sigma_x^2 [\underbrace{0 \; \cdots \; 0}_{N-M} \; \underbrace{g_M^{sig} \; g_{M-1}^{sig} \; \cdots \; g_1^{sig} \; g_0^{sig}}_{M+1} \; \underbrace{0 \; \cdots \; 0}_{N}] \tag{B.19}$$

In addition $\sigma_{\hat{z}}^2$ is given below.

$$\sigma_{\hat{z}}^2 = \sigma_z^2 \sum_{k=1}^{L} w_k w_k^* \tag{B.20}$$

The equation for the MSE in Eq. B.14 is rewritten in vector form.

$$\text{MSE} = \mathbf{c}^H \mathbf{\Phi} \mathbf{c} - \mathbf{c}^H \mathbf{s} - \mathbf{s}^H \mathbf{c} + \sigma_x^2 \tag{B.21}$$

The MSE is minimized with respect to the equalizer tap weights.

$$\frac{\partial}{\partial \mathbf{w}}(\text{MSE}) = 0 \tag{B.22}$$

$$= 2\mathbf{\Phi}\mathbf{c} - 2\mathbf{s} \tag{B.23}$$

Therefore the equalizer tap weights can be found as follows.

$$\mathbf{\Phi}\mathbf{c} = \mathbf{s} \tag{B.24}$$

$$\mathbf{c} = \mathbf{\Phi}^{-1}\mathbf{s} \tag{B.25}$$

The MSE is found in terms of the channel by replacing Eq. B.25 into Eq. B.21.

$$\text{MSE} = \sigma_x^2 - \mathbf{s}^H\mathbf{\Phi}^{-1}\mathbf{s} \tag{B.26}$$

B.2.2 Diversity Combiner Followed by a Decision Feedback Equalizer

We will assume that there are no feedback errors when solving for the equalizer tap weights that minimize the MSE, therefore $\tilde{x}_n^{sig} = x_n^{sig}$. The MSE is given by the equations below.

$$\begin{aligned}
\text{MSE} &= E\left[\left|\hat{x}_n^{sig} - x_n^{sig}\right|^2\right] \qquad \text{(B.27)} \\
&= \sigma_x^2 \underbrace{\sum_{j=-N_1}^{0}\sum_{i=0}^{M}\sum_{q=-N_1}^{0}\sum_{p=0}^{M}}_{i+j=p+q} c_j^* c_q g_i^{sig} g_p^{sig*} - \sigma_x^2 \underbrace{\sum_{j=-N_1}^{0}\sum_{i=0}^{M}}_{i+j=0} c_j^* g_i^{sig} - \\
&\quad \sigma_x^2 \underbrace{\sum_{j=-N_1}^{0}\sum_{i=0}^{M}}_{i+j=0} c_j g_i^{sig*} - \sigma_x^2 \underbrace{\sum_{k=1}^{N_2}\sum_{j=-N_1}^{0}\sum_{i=0}^{M}}_{i+j=k} d_k^* c_j g_i^{sig*} -
\end{aligned}$$

$$\sigma_x^2 \underbrace{\sum_{k=1}^{N_2} \sum_{j=-N_1}^{0} \sum_{i=0}^{M}}_{i+j=k} d_k c_j^* g_i^{sig} + \sigma_x^2 \sum_{k=1}^{N_2} d_k d_k^* + \sigma_x^2 +$$

$$\sigma_x^2 \underbrace{\sum_{j=-N_1}^{0} \sum_{i=0}^{M} \sum_{q=-N_1}^{0} \sum_{p=0}^{M}}_{i+j=p+q} c_j^* c_q \sum_{l=1}^{I} g_i^{int,l} g_p^{int,l*} + \sigma_{\hat{z}}^2 \sum_{j=-N_1}^{0} c_j c_j^* \qquad \text{(B.28)}$$

As before the equation for the MSE is rewritten in vector form. The matrix $\mathbf{\Phi}$ is given by Eq. B.16, but is of size (N_1+1) by (N_1+1). A new matrix $\mathbf{\Gamma}$ is defined as follows.

$$\Gamma_{ij} = \begin{cases} \sigma_x^2 g_{i-j+N_1+1}^{sig} & i-j+N_1+1 \le M \\ 0 & \text{otherwise} \end{cases} \qquad \text{(B.29)}$$

$$\text{where} \quad i = 1, 2, \ldots N_2$$

$$j = 1, 2, \ldots N_1 + 1$$

In addition $\mathbf{c}$, $\mathbf{d}$, and $\mathbf{s}$ are defined as,

$$\mathbf{c} = [c_j] \qquad \text{(B.30)}$$

$$\mathbf{d} = [d_k] \qquad \text{(B.31)}$$

If N_1 is less than or equal to M then $\mathbf{s}$ has the following form.

$$\mathbf{s}^T = \sigma_x^2 [g_{N_1}^{sig} \; g_{N_1-1}^{sig} \; \cdots \; g_1^{sig} \; g_0^{sig}] \qquad \text{(B.32)}$$

Otherwise if N_1 is greater than M then $\mathbf{s}$ has the following form.

$$\mathbf{s}^T = \sigma_x^2 [\underbrace{0 \; \cdots \; 0}_{N_1-M} \; \underbrace{g_M^{sig} \; g_{M-1}^{sig} \; \cdots \; g_1^{sig} \; g_0^{sig}}_{M+1}] \qquad \text{(B.33)}$$

The equation for the MSE in Eq. B.28 is rewritten in vector form.

$$\text{MSE} = \mathbf{c}^H \mathbf{\Phi} \mathbf{c} - \mathbf{c}^H \mathbf{s} - \mathbf{s}^H \mathbf{c} + \sigma_x^2 \mathbf{d}^H \mathbf{d} - \mathbf{d}^H \mathbf{\Gamma}^* \mathbf{c} - \mathbf{c}^H \mathbf{\Gamma}^T \mathbf{d} + \sigma_x^2 \tag{B.34}$$

The vector $\mathbf{w}$ is defined as equal to $[\mathbf{c}\ \mathbf{d}]^T$. We need to rewrite Eq. B.34 such that we can replace of occurrences of $\mathbf{c}$ and $\mathbf{d}$ by $\mathbf{w}$. We begin by rewriting all the vectors in terms of a vectors of size $N_1 + N_2 + 1$ and all the matrices in terms of matrices of size $N_1 + N_2 + 1$ by $N_1 + N_2 + 1$.

$$\begin{aligned} \text{MSE} = {} & [\mathbf{c}^H\ \mathbf{0}] \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \end{bmatrix} - [\mathbf{c}^H\ \mathbf{0}] \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix} - [\mathbf{s}^H\ \mathbf{0}] \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \end{bmatrix} + \\ & [\mathbf{0}\ \mathbf{d}^H] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_x^2 \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{d} \end{bmatrix} - [\mathbf{0}\ \mathbf{d}^H] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{\Gamma}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \end{bmatrix} - \\ & [\mathbf{c}^H\ \mathbf{0}] \begin{bmatrix} \mathbf{0} & \mathbf{\Gamma}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{d} \end{bmatrix} + \sigma_x^2 \end{aligned} \tag{B.35}$$

This is equivalent to the following equation.

$$\begin{aligned} \text{MSE} = {} & [\mathbf{c}^H\ \mathbf{d}^H] \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} - [\mathbf{c}^H\ \mathbf{d}^H] \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix} - [\mathbf{s}^H\ \mathbf{0}] \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} + \\ & [\mathbf{c}^H\ \mathbf{d}^H] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_x^2 \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} - [\mathbf{c}^H\ \mathbf{d}^H] \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{\Gamma}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} - \\ & [\mathbf{c}^H\ \mathbf{d}^H] \begin{bmatrix} \mathbf{0} & \mathbf{\Gamma}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} + \sigma_x^2 \end{aligned} \tag{B.36}$$

The vector $[\mathbf{c}^T \ \mathbf{d}^T]^T$ is replaced with $\mathbf{w}$.

$$\text{MSE} = \mathbf{w}^H \begin{bmatrix} \mathbf{\Phi} & -\mathbf{\Gamma}^T \\ -\mathbf{\Gamma}^* & \sigma_x^2\mathbf{I} \end{bmatrix} \mathbf{w} - \mathbf{w}^H \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix} - [\mathbf{s}^H \ \mathbf{0}]\mathbf{w} + \sigma_x^2 \tag{B.37}$$

The MSE is minimized with respect to the equalizer tap weights to determine the following form for the tap weights.

$$\mathbf{w} = \begin{bmatrix} \mathbf{\Phi} & -\mathbf{\Gamma}^T \\ -\mathbf{\Gamma}^* & \sigma_x^2\mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix} \tag{B.38}$$

Below is the final form for $\mathbf{w}$, found with the inversion identity from [9].

$$\mathbf{w} = \begin{bmatrix} \left(\mathbf{\Phi} - \frac{1}{\sigma_x^2}\mathbf{\Gamma}^T\mathbf{\Gamma}^*\right)^{-1}\mathbf{s} \\ \frac{1}{\sigma_x^2}\mathbf{\Gamma}^*\left(\mathbf{\Phi} - \frac{1}{\sigma_x^2}\mathbf{\Gamma}^T\mathbf{\Gamma}^*\right)^{-1}\mathbf{s} \end{bmatrix} \tag{B.39}$$

The MSE in terms of the channel is found by replacing Eq. B.39 into Eq. B.37.

$$\text{MSE} = \sigma_x^2 - \mathbf{s}^H \left(\mathbf{\Phi} - \frac{1}{\sigma_x^2}\mathbf{\Gamma}^T\mathbf{\Gamma}^*\right)^{-1} \mathbf{s} \tag{B.40}$$

B.3 Probability of Error

The output of the receiver can be rewritten as,

$$\hat{x}_n^{sig} = \sum_{i=-T_1}^{T_2} q_i^{sig} x_{n-i}^{sig} + \sum_{l=1}^{I} \sum_{i=-U_1}^{U_2} q_i^{int,l} x_{n-i}^{int,l} + \hat{z}_n \tag{B.41}$$

The probability of a correct decision is given as,

$$P_c = \frac{4}{2^k}\text{Prob}\{\Re(\hat{x}_n^{sig} - x_n^{sig}) < \sqrt{\frac{\varepsilon}{2}}\} \cdot \text{Prob}\{\Im(\hat{x}_n^{sig} - x_n^{sig}) < \sqrt{\frac{\varepsilon}{2}}\}$$

$$+\frac{4(2^{k/2}-2)}{2^k}\text{Prob}\{\Re(\hat{x}_n^{sig}-x_n^{sig})<\sqrt{\frac{\varepsilon}{2}}\}$$
$$\cdot\text{Prob}\{-\sqrt{\frac{\varepsilon}{2}}<\Im(\hat{x}_n^{sig}-x_n^{sig})<\sqrt{\frac{\varepsilon}{2}}\}$$
$$+\frac{(2^{k/2}-2)^2}{2^k}\text{Prob}\{-\sqrt{\frac{\varepsilon}{2}}<\Re(\hat{x}_n^{sig}-x_n^{sig})<\sqrt{\frac{\varepsilon}{2}}\}$$
$$\cdot\text{Prob}\{-\sqrt{\frac{\varepsilon}{2}}<\Im(\hat{x}_n^{sig}-x_n^{sig})<\sqrt{\frac{\varepsilon}{2}}\} \quad \text{(B.42)}$$

where $\Re(x)$ is the real part of the complex variable x. and $\Im(x)$ is the imaginary part of x. We define p,

$$p = \text{Prob}\{\Re(\hat{x}_n^{sig}-x_n^{sig})\geq\sqrt{\frac{\varepsilon}{2}}\} = \text{Prob}\{\Im(\hat{x}_n^{sig}-x_n^{sig})\geq\sqrt{\frac{\varepsilon}{2}}\} \quad \text{(B.43)}$$

As a first step in finding the upper bound on p, the equation for p is expanded as given below.

$$p = \text{Prob}\left\{\left[\Re(q_0^{sig})-1\right]\Re(x_n^{sig})-\Im(q_0^{sig})\Im(x_n^{sig})+\right.$$
$$\sum_{\substack{i=T_1\\ i\neq 0}}^{T_2}\left[\Re(q_i^{sig})\Re(x_{n-i}^{sig})-\Im(q_i^{sig})\Im(x_{n-i}^{sig})\right]+$$
$$\left.\sum_{l=1}^{I}\sum_{\substack{i=U_1\\ i\neq 0}}^{U_2}\left[\Re(q_i^{int,l})\Re(x_{n-i}^{int,l})-\Im(q_i^{int,l})\Im(x_{n-i}^{int,l})\right]\Re(\hat{z}_n)\geq\sqrt{\frac{\varepsilon}{2}}\right\} \quad \text{(B.44)}$$

By the Chernoff bound [3, 59],

$$p \leq \exp\left(-\zeta\sqrt{\frac{\varepsilon}{2}}\right)E\left[\exp\left(\zeta\Re(\hat{z}_n)\right)\right]\cdot$$
$$E\left[\exp\left(\zeta\left[\Re(q_0^{sig})-1\right]\Re(x_n^{sig})\right)\right]E\left[\exp\left(\zeta\Im(-q_0^{sig})\Im(x_n^{sig})\right)\right]\cdot$$
$$E\left[\exp\left(\zeta\sum_{\substack{i=T_1\\ i\neq 0}}^{T_2}\left[\Re(q_i^{sig})\Re(x_{n-i}^{sig})-\Im(q_i^{sig})\Im(x_{n-i}^{sig})\right]\right)\right]\cdot$$

$$E\left[\exp\left(\zeta\sum_{l=1}^{I}\sum_{\substack{i=U_1\\i\neq 0}}^{U_2}\left[\Re(q_i^{int,l})\Re(x_{n-i}^{int,l})-\Im(q_i^{int,l})\Im(x_{n-i}^{int,l})\right]\right)\right] \quad \text{(B.45)}$$

From [59] we have,

$$E\left[\exp\left(\zeta\Re(\hat{z}_n)\right)\right] = \exp\left(\frac{1}{2}\zeta^2\left(\frac{1}{2}\sigma_{\hat{z}}^2\right)\right) \quad \text{(B.46)}$$

$$E\left[\exp\left(\zeta\Re(q_i)\Re(x_{n-i})\right)\right] \leq \exp\left(\frac{1}{2}\zeta^2\left(\frac{1}{2}\sigma_x^2\right)\left(\Re(q_i)\right)^2\right) \quad \text{(B.47)}$$

$$E\left[\exp\left(\zeta\Im(q_i)\Im(x_{n-i})\right)\right] \leq \exp\left(\frac{1}{2}\zeta^2\left(\frac{1}{2}\sigma_x^2\right)\left(\Im(q_i)\right)^2\right) \quad \text{(B.48)}$$

We substitute Eqs. B.46-B.48 into Eq. B.45 and arrive at the following result.

$$p \leq \exp\left\{-\zeta\sqrt{\frac{\varepsilon}{2}}+\frac{1}{4}\zeta^2\sigma_{\hat{z}}^2+\frac{1}{4}\zeta^2\sigma_x^2\left[1-2\Re(q_0^{sig})+\sum_{l=1}^{I}\sum_{\substack{i=T_1\\i\neq 0}}^{T_2}\left|q_i^{sig}\right|^2 + \sum_{\substack{i=U_1\\i\neq 0}}^{U_2}\left|q_i^{int,l}\right|^2\right]\right\} \quad \text{(B.49)}$$

We now find the value of ζ (greater than zero) that minimizes the right hand side of Eq. B.49. We accomplish this by setting the derivative of p with respect to ζ equal to zero and solving for ζ.

$$\zeta = \frac{\sqrt{\frac{\varepsilon}{2}}}{\frac{1}{2}\sigma_{\hat{z}}^2+\frac{1}{2}\sigma_x^2\left[1-2\Re(q_0^{sig})+\sum_{\substack{i=T_1\\i\neq 0}}^{T_2}\left|q_i^{sig}\right|^2+\sum_{l=1}^{I}\sum_{\substack{i=U_1\\i\neq 0}}^{U_2}\left|q_i^{int,l}\right|^2\right]} \quad \text{(B.50)}$$

We then substitute the result for ζ into Eq. B.49, and arrive at the following result.

$$p \leq \exp\left\{\frac{-\varepsilon/2}{\sigma_{\hat{z}}^2+\sigma_x^2\left[1-2\Re(q_0^{sig})+\sum_{\substack{i=T_1\\i\neq 0}}^{T_2}\left|q_i^{sig}\right|^2+\sum_{l=1}^{I}\sum_{\substack{i=U_1\\i\neq 0}}^{U_2}\left|q_i^{int,l}\right|^2\right]}\right\} \quad \text{(B.51)}$$

Using Eq. B.41 and Eq. 3.13, the MSE can be expressed as,

$$\text{MSE} = \sigma_{\hat{z}}^2 + \sigma_x^2 \left[1 - 2\Re(q_0^{sig}) + \sum_{\substack{i=T_1 \\ i \neq 0}}^{T_2} \left| q_i^{sig} \right|^2 + \sum_{l=1}^{I} \sum_{\substack{i=U_1 \\ i \neq 0}}^{U_2} \left| q_i^{int,l} \right|^2 \right] \tag{B.52}$$

By substituting this result into Eq. B.49, we arrive at the final form of p.

B.4 Zero-Forcing Transmitter

The following equation represents the received signal at time n and at user m, with an MC receiver.

$$\hat{x}_{n,m} = \sum_{k=1}^{L} \sum_{j=-N}^{N} w_{m,j,k}^* y_{n-j,m,k} \tag{B.53}$$

$$= \sum_{k=1}^{L} \sum_{j=-N}^{N} \sum_{\substack{l=1 \\ l \neq m}}^{S} \sum_{p=1}^{L} \sum_{i=0}^{M} w_{m,j,k}^* c_{l,p}^* x_{n-i-j,l} h_{l,m,p,k,i} + \sum_{k=1}^{L} \sum_{j=-N}^{N} w_{m,j,k}^* z_{n-j,m,k} \tag{B.54}$$

where $c_{l,p}$ is the weight of the p^{th} antenna element of the l^{th} transmitting user, $x_{n,l}$ is the data transmitted by user l at time n, $h_{l,m,p,k,i}$ is the channel impulse response transmitting from user l and antenna element p, and received by user m at antenna element k, and $z_{n,m,k}$ is the AWGN at time n at antenna element k of receiving user m. There are S users, all the users have antenna arrays with L elements, and the channel impulse response has a duration of M. The weights of the k^{th} receiving antenna and the j^{th} delay element of the m^{th} receiving user is $w_{m,j,k}$. There are $2N+1$ delay elements in the multitap diversity combiner (MC).

The standard approach to finding the optimal transmitter and receiver weights is minimize the mean-square-error (MSE) with respect to the weights. Therefore the MSE is given as follows.

$$\begin{aligned}
\text{MSE} &= \frac{1}{S}\sum_{m=1}^{S}\Bigg\{\sigma_x^2 \sum_{\substack{l=1\\l\neq m}}^{S}\sum_{p=1}^{L}\sum_{pp=1}^{L} c_{l,p}^{*}c_{l,pp}\sum_{k=1}^{L}\sum_{kk=1}^{L}\sum_{j=-N}^{N}\sum_{jj=-N}^{N} w_{m,j,k}^{*}w_{m,jj,kk} \\
&\quad \underbrace{\sum_{i=0}^{M}\sum_{ii=0}^{M}}_{i+j=ii+jj} h_{l,m,p,k,i}h_{l,m,pp,kk,ii}^{*} \\
&\quad -\sigma_x^2\sum_{p=1}^{L}c_{t,p}^{*}\sum_{k=1}^{L}\sum_{j=-N}^{N} w_{m,j,k}^{*}\sum_{\substack{i=0\\i+j=0}}^{M} h_{t,m,p,k,i} \\
&\quad -\sigma_x^2\sum_{p=1}^{L}c_{t,p}\sum_{k=1}^{L}\sum_{j=-N}^{N} w_{m,j,k}\sum_{\substack{i=0\\i+j=0}}^{M} h_{t,m,p,k,i}^{*} \\
&\quad +\sigma_x^2 + \sum_{k=1}^{L}\sum_{j=-N}^{N} w_{m,j,k}^{*}w_{m,j,k}\Bigg\} \qquad \text{(B.55)}
\end{aligned}$$

where t is the transmitting partner of m. This equation can be written in matrix form.

$$\begin{aligned}
\text{MSE} &= \frac{1}{S}\sum_{m=1}^{S}\Bigg\{\sum_{\substack{l=1\\l\neq m}}^{S}\mathbf{c}^H(l)\mathbf{w}^H(m)\boldsymbol{\Phi}(l,m)\mathbf{w}(m)\mathbf{c}(l) - \mathbf{c}^H(t)\boldsymbol{\Psi}(t,m)\mathbf{w}^{*}(m) - \\
&\quad \mathbf{w}^T(m)\boldsymbol{\Psi}^H(t,m)\mathbf{c}(t) + \sigma_x^2 + \sigma_z^2\mathbf{w}^H(m)\mathbf{w}(m)\Big\} \qquad \text{(B.56)} \\
&= \frac{1}{S}\sum_{m=1}^{S}\Bigg\{\sum_{\substack{l=1\\l\neq m}}^{S}\mathbf{w}^H(m)\mathbf{c}^H(l)\boldsymbol{\Phi}'(l,m)\mathbf{c}(l)\mathbf{w}(m) - \mathbf{w}^H(m)\boldsymbol{\Psi}'(t,m)\mathbf{c}^{*}(t) - \\
&\quad \mathbf{c}^T(t)\boldsymbol{\Psi}'^H(t,m)\mathbf{w}(m) + \sigma_x^2 + \sigma_z^2\mathbf{w}^H(m)\mathbf{w}(m)\Big\} \qquad \text{(B.57)}
\end{aligned}$$

The MSE approaches zero as $\mathbf{c}(l)$ approaches infinity and $\mathbf{w}(m)$ approaches zero. The MSE approaches zero when in the limit the first, second, and third terms of

Eq. B.56 approach σ_x^2. To control this phenomenon, we contain the transmitter weights by a beamforming solution.

First we will find the transmitter weights. The beamforming solution requires that in the direction of the desired receiver the transmitted signal be unity. But in our case we are transmitting to a receiving array, so we transmit unity to the average direction of the receiving array. Therefore,

$$\mathbf{c}^H(t)\mathbf{\Psi}(t,m)\mathbf{1} \;=\; \sigma_x^2 \tag{B.58}$$

The cost function is given by,

$$\frac{1}{S}\sum_{l=1}^{S}\left\{\mathbf{c}^H(l)\left[\sum_{\substack{m=1\\m\neq l}}^{S}\mathbf{w}^H(m)\mathbf{\Phi}(l,m)\mathbf{w}(m)\right]\mathbf{c}(l)-\mathbf{c}^H(l)\mathbf{\Psi}(l,t)\mathbf{w}^*(m)-\right.$$
$$\left.\mathbf{w}^T(t)\mathbf{\Psi}^H(l,t)\mathbf{c}(l)+\sigma_x^2+\zeta\mathbf{c}^H(t)\mathbf{\Psi}(t,m)\mathbf{1}\right\}+\sigma_z^2\mathbf{w}^H(m)\mathbf{w}(m) \tag{B.59}$$

We take the partial derivative of the cost function with respect to $\mathbf{c}(l)$ and set the result to zero. We then solve for $\mathbf{c}(l)$. The result is,

$$\mathbf{c}(l) \;=\; \left[\sum_{\substack{m=1\\m\neq l}}^{S}\mathbf{w}^H(m)\mathbf{\Phi}(l,m)\mathbf{w}(m)\right]^{-1}\left[\mathbf{\Psi}(l,t)\mathbf{w}^*(m)-\zeta\mathbf{\Psi}(t,m)\mathbf{1}\right] \tag{B.60}$$

It can be shown that $\sum_{\substack{m=1\\m\neq l}}^{S}\mathbf{w}^H(m)\mathbf{\Phi}(l,m)\mathbf{w}(m)$ is singular. Therefore this method of finding the transmitter weights has no solution.

An alternate approach for determining the transmitter weights is the zero-forcing solution. First we rewrite Eq. B.54 as follows.

$$\hat{x}_{n,m} \;=\; \sum_{\substack{l=1\\l\neq m}}^{S}\sum_{p=1}^{L}c_{l,p}^*\sum_{i=-N}^{N+M}g'_{l,m,p,i}x_{n-i,l}+\hat{z}_{n,m} \tag{B.61}$$

$$g'_{l,m,p,i} = \sum_{j=\max(0,i-N)}^{\min(M,i+N)} \sum_{k=1}^{L} w^*_{m,j,k} h_{l,m,p,k,j} \tag{B.62}$$

$$\hat{z}_{n,m} = \sum_{k=1}^{L} \sum_{j=-N}^{N} w^*_{m,j,k} z_{n-j,m,k} \tag{B.63}$$

We expand this as follows.

$$\begin{aligned}\hat{x}_{n,m} = & \underbrace{\sum_{p=1}^{L} c^*_{t,p} g'_{t,m,p,0} x_{n,t}}_{\text{transmitted signal}} + \underbrace{\sum_{p=1}^{L} \sum_{\substack{i=-N \\ i\neq 0}}^{N+M} c^*_{l,p} g'_{t,m,p,i} x_{n-i,t}}_{\text{ISI}} + \\ & \underbrace{\sum_{\substack{l=1 \\ l\neq t,m}}^{S} \sum_{p=1}^{L} c^*_{l,p} \sum_{i=-N}^{N+M} g'_{l,m,p,i} x_{n-i,l}}_{\text{Co-channel Interference}} + \underbrace{\hat{z}_{n,m}}_{\text{AWGN}}\end{aligned} \tag{B.64}$$

We need to solve for the transmitter weights such that the ISI and the co-channel interference is nullified. First we rewrite Eq. B.61 in matrix form.

$$\hat{x}_{n,m} = \sum_{\substack{l=1 \\ l\neq m}}^{S} \mathbf{c}^H(l)\mathbf{G}'(l,m)\mathbf{x}(l) + \hat{z}_{n,m} \tag{B.65}$$

For unity transmitted signal in the direction of the desired receiver and to nullify the ISI we have the following for $m = 1 \ldots S$.

$$\mathbf{c}^H(l)\mathbf{G}'(l,m) = \left\{ \begin{array}{cc} \overset{-N}{[0}\cdots 0\overset{0}{1}0\cdots \overset{N+M}{0]} & \text{if } m = t \\ [0\cdots 0] & \text{otherwise} \end{array} \right\} \tag{B.66}$$

Therefore

$$\begin{aligned} & \mathbf{c}^H(l)[\mathbf{G}'(l,1)\cdots\mathbf{G}'(l,t)\cdots\mathbf{G}'(l,S)] = \\ & [\underbrace{0\cdots 0}_{2N+M+1} \cdots \overset{-N}{0}\cdots 0\overset{0}{1}0\cdots \overset{N+M}{0} \cdots \underbrace{0\cdots 0}_{2N+M+1}] \end{aligned} \tag{B.67}$$

We rewrite this equation as follows.

$$\mathbf{c}^H(l)\mathbf{Q}(l) \quad = \quad \mathbf{1}(l) \tag{B.68}$$

As a final solution,

$$\mathbf{c}(l) \quad = \quad \left[\mathbf{Q}(l)\mathbf{Q}^H(l)\right]^{-1}\mathbf{Q}(l)\mathbf{1}(l) \tag{B.69}$$

If $(2N+M+1)S$ is greater than L then Eq. B.68 is underdetermined and we will not completely nullify the interference. In addition, the zero-forcing solution completely neglects the AWGN.

We must now determine the receiver weights. If we derive the values for the weights by MMSE, we will again have the case where the transmitter weights become infinite and the receiver weights become zero. Therefore we will solve for the receiver weights by the beamforming method.

We refer back to Eq. B.57. The first step is to define the following.

$$\mathbf{w}^H(m)\mathbf{\Psi}'(t,m)\mathbf{1} \quad = \quad \sigma_x^2 \tag{B.70}$$

The cost function is given by,

$$\begin{aligned}&\tfrac{1}{S}\textstyle\sum_{m=1}^{S}\left\{\mathbf{w}^H(m)\left[\sum_{\substack{l=1\\l\neq m}}^{S}\mathbf{c}^H(l)\mathbf{\Phi}'(l,m)\mathbf{c}(l)\right]\mathbf{w}(m)-\mathbf{w}^H(m)\mathbf{\Psi}'(t,m)\mathbf{c}^*(t)-\right.\\&\quad\left.\mathbf{c}^T(t)\mathbf{\Psi}^H(t,m)\mathbf{w}(m)+\sigma_x^2+\zeta\mathbf{w}^H(m)\mathbf{\Psi}'(t,m)\mathbf{1}+\sigma_z^2\mathbf{w}^H(m)\mathbf{w}(m)\right\}\end{aligned} \tag{B.71}$$

We take the partial derivative of the cost function with respect to $\mathbf{w}(m)$ and set the result to zero. We then solve for $\mathbf{w}(m)$. The result is,

$$\begin{aligned}\mathbf{w}(m) &= \left[\sum_{\substack{l=1\\l\neq m}}^{S} \mathbf{c}^H(l)\mathbf{\Phi}'(l,m)\mathbf{c}(l) + \mathbf{I}\sigma_z^2\right]^{-1} \cdot \\ &\quad [\mathbf{\Psi}'(t,m)\mathbf{c}^*(t) - \zeta\mathbf{\Psi}'(t,m)\mathbf{1}] \end{aligned} \tag{B.72}$$

To find ζ,

$$\begin{aligned}\mathbf{1}^H\mathbf{\Psi}'^H(t,m)\mathbf{w}(m) &= \mathbf{1}^H\mathbf{\Psi}'^H(t,m)\left[\sum_{\substack{l=1\\l\neq m}}^{S} \mathbf{c}^H(l)\mathbf{\Phi}'(l,m)\mathbf{c}(l) + \mathbf{I}\sigma_z^2\right]^{-1} \cdot \\ &\quad [\mathbf{\Psi}'(t,m)\mathbf{c}^*(m) - \zeta\mathbf{\Psi}'(t,m)\mathbf{1}] = \sigma_x^2 \end{aligned} \tag{B.73}$$

Therefore,

$$\zeta = \frac{\mathbf{1}^H\mathbf{\Psi}'^H(t,m)\left[\sum_{\substack{l=1\\l\neq m}}^{S} \mathbf{c}^H(l)\mathbf{\Phi}'(l,m)\mathbf{c}(l) + \mathbf{I}\sigma_z^2\right]^{-1} \mathbf{\Psi}'(t,m)\mathbf{c}^*(t) - \sigma_x^2}{\mathbf{1}^H\mathbf{\Psi}'^H(t,m)\left[\sum_{\substack{l=1\\l\neq m}}^{S} \mathbf{c}^H(l)\mathbf{\Phi}'(l,m)\mathbf{c}(l) + \mathbf{I}\sigma_z^2\right]^{-1} \mathbf{\Psi}'(t,m)\mathbf{1}} \tag{B.74}$$

Appendix C

Power Control

We begin with Eq. 4.5. The goal is to find the values P_i for all S users which enable the transmission links between transmitting partners to be above the SINR threshold. Therefore we solve the set of linear equations for P_i, following [7]. Eq. 4.5 can be expanded and rewritten as follows.

$$P_t(G_{i,t}^{\mathrm{NISI}} - \gamma_i G_{i,t}^{\mathrm{ISI}}) - \gamma_i \sum_{\substack{j \neq i \\ j \neq t}} P_j G_{i,j} = n_i \gamma_i \qquad i = 1 \ldots S \tag{C.1}$$

This can be restated in vector form.

$$\mathbf{HP} = \mathbf{N} \tag{C.2}$$

where

$$H_{ij} = \begin{cases} G_{i,t}^{\mathrm{NISI}} - \gamma_i G_{i,t}^{\mathrm{ISI}} & \text{if } j = t \\ -\gamma_i G_{i,j} & \text{if } j \neq i \text{ , } j \neq t \\ 0 & \text{if } j = i \end{cases} \tag{C.3}$$

$$N_i = n_i \gamma_i \tag{C.4}$$

However, the matrix $\mathbf{H}$ will have a diagonal of all zeros in this form. Therefore we must interchange the rows between each pair of transmitting and receiving partners. We rewrite the equations as follows.

$$\mathbf{H}'\mathbf{P} = \mathbf{N}' \tag{C.5}$$

where for each pair of users i and t ($i \neq t$),

$$\begin{aligned} H'_{i,j} &= H_{t,j} \\ H'_{t,j} &= H_{i,j} \\ & \quad j = 1 \ldots S \\ N'_i &= n_t \gamma_t \\ N'_t &= n_i \gamma_i \end{aligned} \tag{C.6}$$

We solve for the vector $\mathbf{P}$ by matrix inversion, if the solution exists. Negative values for the transmitting power indicates that no solution exists.

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