Compute the limit: $\lim_{n\to\infty}\int_0^{\frac{\pi}{n}}\sqrt{1+n^2\cos^2(nx)}dx$

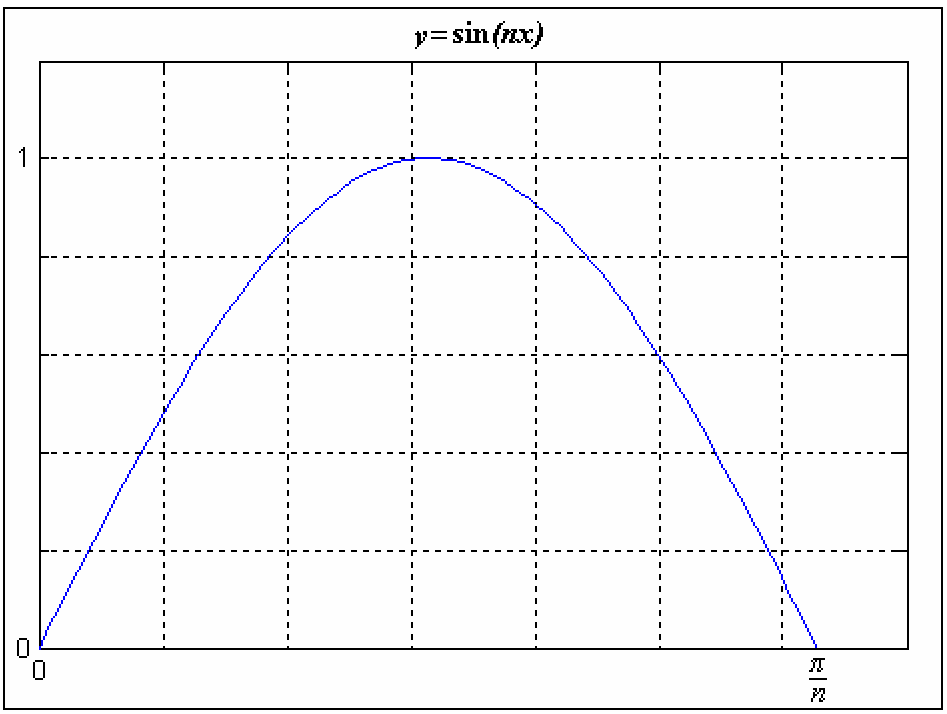
This is nothing more than the arc length of *f(x)* = sin*(nx)* over the interval [0,*π/n*]:

$$L=\int_a^b\sqrt{1+(f'(x))^2}dx$$

$$f'(x)=n\cos(nx)$$

$$L=\int_0^{\frac{\pi}{n}}\sqrt{1+(n\cos(nx))^2}dx=\int_0^{\frac{\pi}{n}}\sqrt{1+n^2\cos^2(nx)}dx$$

The period of sin*(nx)* is 2*π/n*; thus the interval in question is always one half a period.



π/n → 0 as *n* → ∞. This means that one half a period will get squeezed into an infinitely small interval, and thus the arc length gets closer to straight up to 1 and straight back down to 0, making a total of 2.

$$\lim_{n\to\infty}\int_0^{\frac{\pi}{n}}\sqrt{1+n^2\cos^2(nx)}dx=2$$