

$$\int \frac{1}{x} dx$$

$$u = \frac{1}{x} \quad v = x$$

$$du = -\frac{1}{x^2} dx \quad dv = dx$$

$$\int \frac{1}{x} dx = (x) \left(\frac{1}{x} \right) - \int (x) \left(-\frac{1}{x^2} \right) dx \quad (\text{by Integration by Parts})$$

$$\int \frac{1}{x} dx = 1 + \int \frac{1}{x} dx; \quad 0 = 1 \quad \textbf{False}$$

Let's step back to what Integration by Parts is based on:

$$\frac{d}{dx} (x) \left(\frac{1}{x} \right) = \frac{1}{x} dx + \left(-\frac{1}{x} dx \right) = \frac{1}{x} dx - \frac{1}{x} dx = 0 \quad (\text{by Chain Rule})$$

$$\frac{d}{dx} (x) \left(\frac{1}{x} \right) = \frac{d}{dx} 1 = 0$$

$$\frac{d}{dx} (x) \left(\frac{1}{x} \right) - \frac{1}{x} dx = -\frac{1}{x} dx \quad \textbf{True}$$

$$(x) \left(\frac{1}{x} \right) - \int \frac{1}{x} dx = -\int \frac{1}{x} dx; \quad 1 = 0 \quad \textbf{False}$$

The problem must lie within this last step.

$$\text{Does } \int \frac{d}{dx} (x) \left(\frac{1}{x} \right) = (x) \left(\frac{1}{x} \right) \quad ?$$

$$\int \frac{d}{dx} (x) \left(\frac{1}{x} \right) = \int \frac{d}{dx} 1 = \int 0 = C$$

So that last step should be:

$$C - \int \frac{1}{x} dx = -\int \frac{1}{x} dx; \quad C = 0 \quad \textbf{No problem}$$