MATH 5392 – ALGEBRAIC GEOMETRY

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in-class	Let F and G be homogeneous polynomials in $k[X,Y,Z]$ with no common component. Show that $V = \mathrm{V}(F,G,Z) \subset \mathbb{P}^2$ is finite.

Assume to the contrary that it is infinite. Denote an average point in $\mathbb{P}^2$ as $[X : Y : Z]$. For a point in V, we know that $Z = 0$ so that one of X or Y must be nonzero. If V is infinite, then there must either occur infinitely many points with X nonzero or infinitely many with Y nonzero (if not both). Suppose without loss of generality that there are infinitely points with X nonzero. Since $[X : Y : 0] = [1 : \frac{Y}{X} : 0]$, we have that $F(1, \frac{Y}{X}, 0)$ is a polynomial which is zero for infinitely many values of $\frac{Y}{X}$. Similarly we can deduce the same for G. This implies that these are the zero polynomial, which then implies that Z divides the original polynomials. This contradiction proves the proposition.

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