MATH 5392 – ALGEBRAIC GEOMETRY

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6.7 Prove that the intersection of a hypersurface $V \subset \mathbb{A}^n$ (not a hyperplane) with the tangent hyperplane T_pV is singular at p.

Let $V = V(F)$ and let $G = \sum_{i=1}^n \frac{\partial F}{\partial X_i}(p) \cdot (X_i - a_i)$. Then the intersection in question is $V(F, G)$. Excepting the hyperplane case, we can look at the Jacobian

$$\frac{\partial(F, G)}{\partial(X_1, \ldots, X_n)}$$

By the very definition of G the two by two minors of this matrix are all zero at p, whence it is not of full rank at p. From this it follows that p is a singular point in the intersection.

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