GTR is founded on a Conceptual Mistake

And hence Null and Void

A critical review of the fundamental basis of General Theory of Relativity shows that a conceptual mistake has been made in the basic postulate of General Relativity through violation of the principle of 'invariance' of space points. By linking the metric of space with the gravitational field, the physical space is implied to be deformable and subjected to an incompatible set of strain components. Since the focus of our attention in this article is going to be on the Conceptual Mistake made in the very foundation of General Theory of Relativity, let us begin our analysis with a review of fundamental definitions of relevant basic concepts**.**

Definitions of relevant Basic Concepts

- **Coordinate Systems:**

  The cardinal idea responsible for the invention of coordinate systems by Descartes consists of the assumption that to each real number there corresponds a unique point on a straight line. We choose a straight line X and a point O on it, which we call the origin. We choose a point A and call the length of the line segment OA, the unit length. Next we pick up any point P on this line X, as shown in the figure and take the ratio of the lengths of the line segments OP and OA. Let this ratio OP/OA be equal to x. The number x is called the coordinate of P.

  ............O...........A..P............................> X

  The association of the set of points P on coordinate line X with the set of real numbers x, constitutes a coordinate system of the one-dimensional SPACE, once the notion of certain 'unit length' OA has been defined.

- **Coordinate Space:**

  The coordination of the set of points lying in the plane with sets of real numbers is accomplished by taking two orthogonal coordinate lines X^1 and X^2 in that plane and defining the notion of unit lengths on each of them. With each point P in the plane of the coordinate axes we associate an ordered pair of real numbers (x^1,x^2) termed coordinates of that point. The one-to-one correspondence of ordered pairs of numbers with the set of points in the plane X^1X^2 is the coordinate system of the two-dimensional SPACE consisting of points in the plane. The extension of this representation to points in a 3-dimensional space is obvious. With predefined notion of unit length, the essential feature of it is the concept of one-to-one correspondence of points in SPACE with the ordered sets of real numbers. Spaces, where it is possible to construct a coordinate system such that the length of a line segment is given by the formula of Pythagoras, are called Euclidean spaces. In these spaces the notion of displacements is fundamental.

- **Vector Components and Transformations:**

In a three-dimensional space, let us choose three orthonormal vectors a_1, a_2, a_3, as our coordinate vectors. In this case any vector X has the representation,

$$X = x^1 a_1 + x^2 a_2 + x^3 a_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where (x^1, x^2, x^3) are called the physical components or measure numbers of X and the vectors a_1, a_2, a_3 contain the notion of ' unit lengths' along the coordinate lines. If however, the coordinate vectors $\mathbf{a_1}$, $\mathbf{a_2}$, $\mathbf{a_3}$, are not of unit length then (x^1, x^2, x^3) will represent different measure numbers or contravariant components of X. The corresponding physical projections of X along coordinate directions, will be given by the product of such contravariant components with their base vectors as $x^1 * |a_1|$; $x^2 * |a_2|$ and $x^3 * |a_3|$ respectively. The matrix A in the equation X' = A X can be interpreted as an operator which converts a vector X into another vector X' through a transformation of its components. We may interpret the resulting vector X' as a deformed vector produced by the operator A. It is very important to note here that the deformation of vector X can be brought about either through the variation of its components x^1, x^2, x^3 by some operator A or through the variation of the set of base vectors a_1, a_2 and a_3 by the transformation of reference coordinate system.

- **Invariance of Space Points:**

In particular, we may deal with such transformation of components x^i and base vectors a_i such that the vector X itself remains invariant. The concept of invariance of mathematical objects, called vectors and tensors, under coordinate transformations, permeates the whole structure of tensor analysis. We shall suppose that a point is an invariant. In a given reference frame a point P is determined by a set of coordinates x^i. If the coordinate system is changed, the point P is described by a new set of coordinates y^i, but the transformation of coordinates does nothing to the point itself. A set of points, such as those forming a curve or surface, is also invariant. The curve may be described in a given coordinate system by an equation, which usually changes its form when the coordinates are changed, but the curve itself remains unaltered, invariant. Similarly a triply infinite set of points, constituting a 3-D space , may also be considered invariant if an infinitesimal separation distance ' ds' between any pair of neighboring space points remains invariant under admissible coordinate transformations. **The notion of invariance of the arc element ' ds' in all admissible coordinate transformations is most crucial in the formulation and efficacy of tensor analysis.**

However, it is extremely important to understand that the ' invariance' of mathematical objects, like vectors, is only in respect of coordinate transformations. In any particular coordinate system when we define certain vector or tensor we are free to assign any value to it, but once assigned that value will remain invariant under all admissible coordinate systems. Of course in any particular coordinate system, we are always free to redefine that vector or tensor or to re-assign any other value to it on physical considerations.

Coordinate Transformations and the Metric Tensor $[g_{ij}]$

Let us consider a displacement vector (dy^i) determined by a pair of points P(y) and P' (y+dy) referred to orthogonal Cartesian coordinates y^j. The square of distance between points P and P' is given by the formula of Pythagorasas

$$(ds)^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2 \quad \text{................. (2)}$$

where ds is called the element of arc. A change in coordinate system from y^i to x^i given by the transformation relations

$$y^i = y^i(x^1, x^2, x^3) \quad \text{.................... (3)}$$

permits us to write the relation (2) as

$$ds^2 = (g_{ij})(dx^i)(dx^j) \quad \text{.................... (4)}$$

with usual summation over indices i & j from 1 to 3 and where the metric tensor coefficients are given by the partial derivatives of y^i as,

$$g_{ij}(x) = (\partial y^k/\partial x^i)(\partial y^k/\partial x^j) \qquad \text{sum for } k = 1 \text{ to } 3 \quad \text{........... (5)}$$

Here the equations (4) and (5) will jointly ensure that the length of the arc element ds remains invariant with the transformation of coordinates (3). If we do not ensure the invariance of arc element during coordinate transformations (say, by arbitrarily changing the notion of unit length or the metric of space), the whole structure of tensor analysis may crumble. Any new coordinate system will have its corresponding metric coefficients uniquely defined through relations of the type (3) and (5). In orthogonal coordinate systems, the value of three metric coefficients g_{11}, g_{22}, g_{33} determines the magnitude of corresponding base vectors $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$ as $\mathbf{a_1}.\mathbf{a_1} = g_{11}$; $\mathbf{a_2}.\mathbf{a_2} = g_{22}$; $\mathbf{a_3}.\mathbf{a_3} = g_{33}$. And the square of arc element is,

$$ds^2 = g_{11}*(dx^1)^2 + g_{22}*(dx^2)^2 + g_{33}*(dx^3)^2 \quad \text{.....................(6)}$$

Differential Scaling Effects of Metric Coefficients

Let us consider a 4-dimensional space-time manifold covered with spherical polar orthogonal coordinates system with its origin at point O and the coordinate parameters y^i of an arbitrary point P given by,

$$y^1 = R\ ; \quad y^2 = \theta\ ; \quad y^3 = \phi \ \text{ and } \ y^4 = T$$

such that the invariant arc element dS is given by,

$$dS^2 = (dR)^2 + (R.d\theta)^2 + (R.\sin(\theta).d\phi)^2 + (\iota c.dT)^2 \quad \text{.....................(7)}$$

The spatial metric coefficients of this coordinate system can be written as,

$$g_{11} = 1\ ; \ g_{22} = R^2\ ; \ g_{33} = R^2.\sin^2\theta\ ; \ g_{44} = 1 \quad \text{................... (8)}$$

And the magnitude of corresponding spatial base vectors is given by,

$$|a_1| = 1 \ ; \quad |a_2| = R \ ; \quad |a_3| = R.\sin(\theta) \ ; \quad |a_4| = 1 \quad \text{................... (9)}$$

So that the arc element vector **dS** may be given by,

$$\mathbf{dS} = dR\ \mathbf{a_1} + d\theta\ \mathbf{a_2} + d\phi\ \mathbf{a_3} + \iota\ c.dT\ \mathbf{a_4} \quad \text{..................... (10)}$$

Now consider a coordinate transformation from coordinate system y^i to another coordinate system $x^i(r, \theta, \phi, t)$ through transformation relations of the type (3) as,

$$R = 2r ; \quad \theta = \theta ; \quad \phi = \phi ; \quad T = t$$

So that the invariant arc element dS in (x^i) space will be given by,

$$dS^2 = \{(2dr)^2 + (2r.d\theta)^2 + (2r.\sin(\theta).d\phi)^2\} - (c.dt)^2 \qquad ....(11)$$

The spatial metric coefficients of this coordinate system x^i can now be written as,

$$h_{11} = 4 ; \quad h_{22} = 4\, r^2 ; \quad h_{33} = 4\, r^2.\sin^2\theta \qquad .... (12)$$

And the magnitude of corresponding spatial base vectors is given by,

$$|a_1| = 2 \;\; ; \;\; |a_2| = 2\, r \;\; ; \;\; |a_3| = 2\, r.\sin(\theta) \qquad .... (13)$$

An infinitesimal arc element vector **dS** will therefore be given by,

$$\mathbf{dS} = dr\, \mathbf{a_1} + d\theta\, \mathbf{a_2} + d\phi\, \mathbf{a_3} + \iota\, c.dt\, \mathbf{a_4} \qquad .... (14)$$

Here the coordinate parameter r does not represent the length of the line segment OP. The fact that the magnitude of this coordinate parameter r has been reduced to R/2, can not be construed to imply that the coordinate space has been shrunk by virtue of this coordinate transformation or by virtue of changed values of the associated metric coefficients. It only implies that the coordinate space has been scaled down by virtue of changed values of the associated metric coefficients. Here too, the length of the line segment OP will be given by the product of the coordinate parameter r and the length of the base vector a_1, that is by r^*a_1, which remains invariant.

Riemannian Space and the Curvature Tensor

Spaces, where it is not possible to construct a coordinate system such that the length of a line segment is given by the formula of Pythagoras, are called Riemannian spaces. In other words, **the metric coefficients $g_{ij}(x)$ of a Riemannian space cannot be transformed into constant components h_{ij} by any admissible coordinate transformation.** But a necessary and sufficient condition that a symmetric tensor $g_{ij}(x)$ with $|g_{ij}| \neq 0$, reduce under a suitable transformation of coordinates to a tensor h_{ij}, where the coefficients h_{ij}'s are constants, is that the Riemann-Christoffel tensor formed from the g_{ij}'s be a zero tensor. The Riemann-Christoffel tensor or simply the Riemann tensor is given by the relation,

$$R^i_{jkl} = \partial/\partial x^k\, \Gamma^i_{jl} - \partial/\partial x^l\, \Gamma^i_{jk} + \Gamma^i_{\alpha k}\Gamma^\alpha_{jl} - \Gamma^\alpha_{jk}\Gamma^i_{\alpha l} \qquad \text{summation over } \alpha \qquad .... (15)$$

where Γ^i_{jk} is a Christoffel symbol of second kind. Therefore, the notion of Riemannian space may be directly associated with the non-zero value of Riemann tensor formed from the components of the metric tensor. However, non-zero value of the Riemann tensor is known to represent the curvature of two-dimensional parametric surfaces. A non-zero value of Riemann tensor in 3-D space or 4-D space-time manifolds is therefore associated with the notion of 'curvature' of the corresponding space or space-time manifolds. The Riemann tensor is also referred to as the curvature tensor of the concerned space or space-time manifold. **Hence the Riemannian space can be identified with finite non-zero value of the Riemann tensor .**

Notion of 'Space Points' and 'Space-time Points'

The notion of a point in a 3 D physical space represents a space point, whereas the notion of an event in 4 D reference framework represents a space time point. We have to be careful about this description when considering a space time interval for studying a time varying phenomenon. As per the most fundamental concept of tensor analysis, a space point is an invariant. If we identify two neighboring space points as P_1 and P_2, they will remain points P_1 and P_2 for all time to come and their separation distance 'ds' will remain time invariant constant. In a 4 D space time reference framework, these points will constitute time like traces P1' and P2' . At any given instant of time t_1, a spatial section of the time like traces P1' and P2' in the 4 D reference frame work, will represent the original space points P_1 and P_2 with their separation distance 'ds'. The most fundamental invariant characteristics of space points do not vanish merely by their representation in a 4 D reference framework. There is a correspondence between the 3 D space points and the time like parallel traces in the 4 D space time reference framework. **The notion of invariance of space points signifies the time invariant constancy of their mutual separation distances and this notion is also used for defining admissible coordinate transformations in 3 D space.** However, the notion of invariance of events or space time points does not imply any time invariance; but a similar notion of invariance of space time interval between two space time points or events is used for defining admissible coordinate transformations in 4 D space time manifold or reference frame work.

Main Postulate of General Relativity

The main postulate of General Relativity is that the gravitational phenomenon can be satisfactorily represented by ' suitably adjusting' the metric properties of the space time manifold. For this the metric coefficients of the space time manifold are required to satisfy a set of partial differential equations (EFE) involving energy-momentum tensor, whereby the non-rectilinear trajectories of mass particles will transform into geodesics. Thus the study of dynamical trajectories of mass particles in a gravitational field will reduce to the study of geodesics in the space time manifold defined by specified metric coefficients. After doing so, the main postulate could be extended to imply that the gravitational field itself 'somehow' modifies the metric coefficients of space-time manifold such that the trajectories of mass particles ' naturally' turn out to be geodesics.

Line element 'ds' and the Space Metric

The most fundamental concept underlying all the basic notions of space is that of the absolute invariance of space points. Say, in any particular coordinate system X with origin at point O, let us consider a particular space point P_1 with coordinates (x^i). If P_2 is another space point in the neighborhood of P_1, then an infinitesimal separation distance ds between the points P_1 and P_2 is given by:

$$(ds)^2 = g_{ij}\, dx^i\, dx^j \qquad \text{..................... (16)}$$

where g_{ij} are the metric coefficients in coordinate system x^i. The invariance of space points P_1, P_2 etc. implies that ' ds' will remain constant, even when the coordinate system is changed (or more correctly ' transformed') from X to say Y. Obviously, from equation (16), once the coordinate parameters x^i are changed to y^i, the metric coefficients g_{ij} must also transform to say h_{ij} to ensure the invariance of ' ds' . As such,

$$(ds)^2 = h_{ij}\, dy^i\, dy^j \qquad \text{.................... (17)}$$

Further, it can be easily seen from equations (16) and (17) that the transformation of metric coefficients $g_{ij}(x)$ to the coefficients $h_{ij}(y)$ has to be intimately related to the transformation of coordinate parameters x^i to the parameters y^i, so as to ensure the invariance of ds. Such transformations that ensure the invariance of ds and hence the invariance of space points in general, are said to be admissible transformations. Obviously, if the coefficients g_{ij} constitute the metric of Euclidean space, then all other coefficients h_{ij} etc. obtained through any admissible transformation of coordinates will also represent the metric of same Euclidean space.

On the other hand let us consider an arbitrary change in metric coefficients $g_{ij}(x)$ in equation (16) to $g1_{ij}(x)$. Here, by an arbitrary change we mean a change that is brought about on any considerations other than through an admissible transformation of coordinates. From equation (16), it can be easily seen that an arbitrary change in metric coefficients $g_{ij}(x)$ to say $g1_{ij}(x)$, without any corresponding change in coordinate parameters x^i, will lead to a change in separation distance ' ds' between points P_1 and P_2 to say ds_1. Therefore,

$$(ds_1)^2 = g1_{ij}\, dx^i\, dx^j \qquad \text{................. (18)}$$

Obviously, from equations (16) and (18), ' ds_1' cannot be equal to ' ds' until and unless $g1_{ij}(x)$ is equal to $g_{ij}(x)$. The 'arbitrarily changed' coefficients $g1_{ij}(x)$ can be associated with the metric of a Riemannian space, whereas the original coefficients $g_{ij}(x)$ represented the metric of Euclidean space. Hence, it may be concluded that whenever the metric $[g_{ij}(x)]$ of Euclidean space is changed (through whatever means - other than admissible transformations) to the metric $[g1_{ij}(x)]$ of Riemannian space, the separation distance ' ds' between two neighboring points P_1 and P_2 given by equation (16) will change to the separation distance ' ds_1' as given by equation (18).

Any change in separation distance ds between space points P_1 & P_2 to ds_1 will imply a **relative displacement** between neighboring points P_1 & P_2, **leading to the concept of deformable space** where the space points are no longer held invariant. This notion of relative displacement (with displacement vector **U**) will be applicable at all space points where the metric coefficients $g_{ij}(x)$ are changed to $g1_{ij}(x)$.

GR Postulate implies Deformable Space

However, as per General Relativity, it is a fundamental postulate of GR that a gravitating body with its gravitational field changes the metric $[g_{ij}(x)]$ of Euclidean space to the metric $[g1_{ij}(x)]$ of Riemannian space in accordance with Einstein's Field Equations. With this change in the metric of space, the separation distance 'ds' between any pair of neighboring space points P_1 and P_2 will also change to 'ds_1' as given by equations (16) and (18). That means all space points within the region of gravitational field will experience **relative displacements** (with displacement vector **U**) leading to the development of **deformation** of space in that region. Hence, in essence, the fundamental postulate of GR implies the deformation of space in the region of gravitational field.

This displacement vector field **U** will naturally lead to the development of a strain field throughout the region of space under consideration. In the study of a continuous medium, we consider a continuum of identifiable material points, mathematical representation of which constitutes a manifold. The deformation characteristics of such a continuous media are studied through the study of changes in metric coefficients of the manifold. In fact, subtracting (16) from (18) we get:

$$(ds_1)^2 - (ds)^2 = \{g1_{ij} - g_{ij}\}\, dx^i\, dx^j = 2.e_{ij}\, dx^i\, dx^j \qquad \text{..............} (19)$$

where e_{ij} represent the components of a strain tensor expressed as functions of coordinate parameters x^i.

Gravitational Field induced Strain Components

Let us consider a spherical polar coordinate system with origin at point O and the coordinate parameters r, θ and φ. For a spherically symmetric mass particle M of physical radius r_0, located at the origin O of the coordinate system, if we consider the gravitational field in its vicinity (i.e. $r > r_0 > 0$), the radial metric coefficient g_{rr} is given by the Schwarzschild solution as:

$$g_{rr} = 1/(1 - 2GM/c^2r) \;;\quad g_{\theta\theta} = r^2 \;;\quad g_{\phi\phi} = r^2.\mathrm{Sin}^2(\theta) \qquad \text{............} (20)$$

Thus the radial metric coefficient g_{rr} at any particular space point $P_1(r, \theta, \phi)$ can be taken as a function of M, that is $g_{rr}(M)$. Its value in the region under consideration is always greater than unity for M>0. The arc element or the separation distance ds_r between two neighboring space points P_1 and P_2 in this region will be given by:

$$(ds_r)^2 = g_{rr}\,(dr)^2 + g_{\theta\theta}\,(d\theta)^2 + g_{\phi\phi}\,(d\phi)^2$$
$$= (1/(1 - 2GM/c^2r)).(dr)^2 + r^2.(d\theta)^2 + r^2.\mathrm{Sin}^2(\theta).(d\phi)^2 \qquad \text{..........} (21)$$

As a part of detailed mathematical analysis, let us study the variation of ds_r as a function of M. Holding all other parameters constant, any change in M (say from M to M+dM), will induce a corresponding change in radial metric coefficient g_{rr} which will lead to the corresponding change in the separation distance ds_r between the two neighboring space points P_1 and P_2 under consideration. Therefore ds_r can now be considered as a function of M, i.e. $ds_r(M)$. Let us give a general notation $[g_{ij}(M)]$ to the metric tensor with its coefficients $g_{rr}(M)$, $g_{\theta\theta}$ and $g_{\phi\phi}$ in coordinate system (r, θ, φ). With this general notation $[g_{ij}(M)]$ for the metric tensor, equation (21) for the separation distance $ds_r(M)$ between two neighboring space points P_1 and P_2 in this region, can be re-written in a more compact notation, in line with equations (16) and (18) above, as:

$$\{ds_r(M)\}^2 = g_{ij}(M)\, dx^i\, dx^j \qquad \text{.....................} (22)$$

where x^i, x^j refer to coordinate parameters (r, θ, φ).

In the small region under consideration, let us study the CHANGE in the separation distance ds_r between two neighboring space points P_1 and P_2, when mass of the body located at the origin of reference coordinates (r, θ, ϕ) is CHANGED from M to M+dM (through whatever means). The changed separation distance will again be given by equation (22) as,

$$\{ds_r(M+dM)\}^2 = g_{ij}(M+dM)\, dx^i\, dx^j \qquad \text{.................. (23)}$$

From equations (19), (22) and (23), we can compute the CHANGE or the VARIATION in the separation distance ds_r between two neighboring space points P_1 and P_2, when M gets changed to M+dM.

$$\{ds_r(M+dM)\}^2 - \{ds_r(M)\}^2 = \{ g_{ij}(M+dM) - g_{ij}(M)\} dx^i\, dx^j = 2.e_{ij}\, dx^i\, dx^j \qquad \text{.........(24)}$$

Whenever $ds_r(M+dM)$ is different from $ds_r(M)$, a strain field $e_{ij}(x)$ can be associated with all space points, as induced by the change in mass of the gravitating body from M to M+dM or in other words, as induced by a change in the metric tensor from $[g_{ij}(M)]$ to $[g_{ij}(M+dM)]$. In the reference coordinate system (r, θ, ϕ) considered above, let us substitute the value of metric coefficients $g_{ij}(M)$ and $g_{ij}(M+dM)$ in equation (24) to get,

$$2.\, e_{rr} = \{g_{rr}(M+dM) - g_{rr}(M)\} \qquad \text{.................... (25)}$$

$$\text{and} \qquad e_{\theta\theta} = e_{\phi\phi} = 0 \qquad \text{.................... (26)}$$

Substituting the simplified value of g_{rr} from equation (20), we get (**an incremental value of e_{rr}** induced by dM)

$$e_{rr} = \{(1 + 2G(M+dM)/c^2r) - (1 + 2GM/c^2r)\}/2 = G.dM/c^2r \qquad \text{................. (27)}$$

This shows that the total radial strain e_{rr} induced in space by the gravitational field, is directly proportional to the mass M of the gravitating body. All other strain components ($e_{\theta\theta}$ & $e_{\phi\phi}$) are zero. This set of strain components constitutes the strain field induced in the region of space influenced by the gravitational field of M.

GR induced Elasticity of Riemannian 3-D Space

We have seen earlier that as the gravitational field develops or varies in a region of space under consideration, the radial metric coefficient g_{rr} and hence the radial strain component e_{rr} varies accordingly, leading to the radial deformation of space. This radial deformation will keep developing or increasing with the development of gravitational field in that region. The radial deformation of Riemannian space induced by a developing gravitational field, through the variation of its metric tensor, is a reversible phenomenon. That means a developing gravitational field increases the radial metric coefficient g_{rr}, thereby increasing the radial spacing between concentric spherical surfaces and thus leading to the increase in radial strain or deformation. But when the gravitational field is reduced back to the initial state, the radial strain will also get reduced to the initial state. This reversible characteristic of the induced radial strain field in response to the external influence of gravitational field, actually implies **an elastic response** of the Riemannian space!! This implied notion of elasticity property of space is further strengthened with the associated notions of the 'energy' of gravitational field. Hence, we might view this revised notion of space, which is defined to be a continuum of space points, as an Elastic Space.

Mutual incompatibility of the Strain Components

The existence of radial strain components e_{rr} given by equation (27), with all other components being zero, violates the compatibility conditions for the strain components. In order to illustrate and highlight this problem, let us consider the relative displacement vector **U** that gives rise to the strain components e_{rr}, $e_{\theta\theta}$ and $e_{\phi\phi}$ of equations (26, 27). If u^r is the radial component of the displacement vector **U**, then the strain components dependent on u^r are given by,

$$e_{rr} = \partial u^r/\partial r \quad ; \quad e_{\theta\theta} = u^r/r \quad \text{and} \quad e_{\phi\phi} = u^r/r \qquad \ldots\ldots\ldots\ldots\ldots\ (28)$$

Obviously, if the radial strain component e_{rr} is non-zero, the radial displacement component u^r must be non-zero. But once the radial displacement component u^r is non-zero, the tangential strain components $e_{\theta\theta}$ and $e_{\phi\phi}$ cannot be zero. This precisely is the incompatibility of the strain components e_{rr}, $e_{\theta\theta}$ and $e_{\phi\phi}$ induced by the static gravitational field of a spherically symmetric gravitating body of mass M. This incompatibility is not limited to the strain components induced by the Schwarzschild metric of spherically symmetric, static gravitational fields but is applicable to all strain components induced by the Riemannian metric obtained from EFE. In fact one of the essential requirements imposed by the standard compatibility conditions on strain components e_{ij} is that the Riemann tensor composed from e_{ij} must be a zero tensor. This can be true only if both metrics of equation (19), namely $g1_{ij}$ and g_{ij} are Euclidean which however contradicts the basic postulate of General Relativity. Therefore, the specification of metric coefficients (20) as per the Schwarzschild solution is physically invalid and unacceptable.

Physical Invalidity of General Theory of Relativity

In GR, the ' structure' of real physical space has been tampered with imprudently by hypothesizing that the metric coefficients of space are affected by the presence of gravitational field as per the Einstein's Field Equations. The Einstein's Field Equations require the metric of space under gravitational influence to be inherently Riemannian. However, as seen above, when the metric of space is 'changed' from Euclidean to Riemannian, the 'induced' deformation of space gives rise to a mutually incompatible set of strain components leading to discontinuities and voids that are physically invalid and unacceptable. The compatibility conditions of strain components require the metric of space to be Euclidean even under the influence of a gravitational field.

Therefore, the main postulate of GR and the Einstein's Field Equations are found to be physically invalid, firstly on account of the 'deformation of space' induced by the Riemannian metric and secondly on account of the violation of essential compatibility conditions for the induced strain components in the space thus 'deformed'. The General Theory of Relativity is thus physically invalid, null and void, irrespective of any claimed utility, application or validation of this 'theory'.

G. S. SANDHU

Appendix 'A'

Review of Einstein's Original GR Postulate

[Relevant Excerpts from an old book 'THEORY OF RELATIVITY' (PART IV – GTR) by W. PAULI, translated from the German by G. FIELD]]

"As soon as the physical deductions from the special theory of relativity had reached a certain stage, Einstein at once attempted to extend the relativity principle to reference systems in non-uniform motion. He postulated that the general physical laws should retain their form even in systems other than Galilean. This was made possible by the so-called principle of equivalence. The problem next arose of how to set up such a theory which was to be based on the principle of equivalence and which would also apply to non-homogeneous gravitational fields. If the square of the line element is transformed into an arbitrary curvilinear space-time coordinate system, it becomes a quadratic form in the coordinate differentials, with ten coefficients g_{ik}. The gravitational field is now determined by this ten-component tensor of the g_{ik}, and no longer by the scalar light velocity. At the same time the equation of motion of a particle, the energy-momentum law and the electromagnetic field equations for the vacuum, were all given a definite, generally covariant, form by introducing the g_{ik}. Only the differential equations for the g_{ik} themselves were not generally covariant yet. In a subsequent paper (1914), Einstein tried to establish these differential equations in a more rigorous manner and he even believed to have proved that the equations that determine the g_{ik} themselves could not be generally covariant. In the year 1915, however, he realized that his gravitational field equations were not uniquely determined by the invariant-theoretical conditions, which he had formerly laid down for them. To restrict the number of alternatives, he reverted to the postulate of general covariance, which he had previously 'abandoned only with a heavy heart'. Making use of Riemann' s theory of curvature, he in fact succeeded in setting up generally covariant equations for the g_{ik} themselves, which met all the physical requirements.

Originally, the principle of equivalence had only been postulated for homogeneous gravitational fields. For the general case, it can be formulated in the following way: For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either on the motion of particles or any other physical processes. In short, in an infinitely small world region, every gravitational field can be transformed away. (We can think of the system K_0 in terms of a small box freely falling under gravity.) It is clear that this "transforming away" is only possible because the gravitational mass is always equal to the inertial mass. It is evidently natural to assume that the special theory of relativity should be valid in K_0. All its theorems have thus to be retained, except that we have to put the system K_0, defined for an infinitely small region, in place of the Galilean coordinate system. All systems K_0, which are derived from each other by a Lorentz transformation, are on the same footing. In this sense we can therefore say that the invariance of the physical laws under Lorentz transformations also persists in infinitely small regions. We can now associate with two infinitely close point events a certain measurable number, their distance ds. For this, we only need to transform away the gravitational field and then form, in K_0 the quantity,

$$ds^2 = (dX_1)^2 + (dX_2)^2 + (dX_3)^2 - (dX_4)^2 \qquad \text{............ (A1)}$$

Let us now consider some other coordinate system K in which the values of the coordinates x^1, ..., x^4 are assigned to the world points in a completely arbitrary way, apart from the conditions of uniqueness and continuity. At each space-time point, the corresponding differentials dX_i will then be linear homogeneous expressions in the dx^k, and the line element ds^2 will be transformed into the quadratic form,

$$ds^2 = g_{ik}\, dx^i\, dx^k \qquad \text{................ (A2)}$$

where the coefficients g_{ik} are functions of the coordinates. It is moreover obvious that for a transition to new coordinates, the g_{ik} transform in such a way that ds^2 remains invariant. The situation is thus completely analogous to that obtaining in the geometry of non-Euclidean multidimensional manifolds. The system K_0 in a freely falling box takes the place of the geodesic system; the g_{ik} in it are constant, so long as their second derivatives can be neglected and the line element is of the form (A1) up to terms of second order. The totality of the g_{ik} values at all world points will be called the G-field. The equation of motion of a particle, which is subjected to no forces other than gravity, can now be set up very easily. The world line of such a particle is a geodesic line,

$$d^2x^i/ds^2 + \Gamma^i{}_{jk}\, dx^j/ds\; dx^k/ds = 0 \qquad \text{................ (A3)}$$

where $\Gamma^i{}_{jk}$ is a Christoffel symbol of second kind. For in system K_0 the particle is in a rectilinear uniform motion at a given moment, i.e. $d^2x^i/ds^2 = 0$, which is at the same time the system of equations of the geodesic line in K_0. Now the statement, that the world line of a particle is a geodesic line, is invariant and therefore holds generally. (We have assumed here, however, that the second derivatives of the g_{ik} with respect to the coordinates do not appear in the equation of motion of the particle.) The validity of this simple theorem is not surprising. It is just due to the fact that the line element was defined in such a way that the world line of a particle becomes a geodesic line. **We thus see that the ten tensor components g_{ik} in Einstein's theory take the place of the scalar Newtonian potential Φ; the components $\Gamma^i{}_{jk}$ formed from their derivatives, determine the magnitude of the gravitational force.**

There is, however, also a third way in which the G-field can be measured. With the help of measuring rods (or better, measuring threads) and clocks we could determine, for a given coordinate system, the dependence of the magnitude ds of the line element on the coordinate differentials dx^k along all world lines originating from an arbitrary point. From this the G-field follows immediately. It thus characterizes not only the gravitational field but also the behavior of measuring rods and clocks, i.e. the metric of the four-dimensional world, which contains the geometry of ordinary three-dimensional space as a special case. This fusion of two previously quite disconnected subjects - metric and gravitation - must be considered as the most beautiful achievement of the general theory of relativity. The motion of a particle under the sole influence of gravity can now be interpreted in the following way: The motion of the particle is force-free. It is not rectilinear and uniform because the four-dimensional space-time continuum is non-Euclidean and because in such a continuum a rectilinear uniform motion has no meaning and has to be replaced by motion along a geodesic line. This fusion of gravity and metric leads to a satisfactory solution not only of the gravitational problem, but also of that of geometry. The general theory of relativity now allows us immediately to make a general statement: **Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter.**"