Title : ELASTIC CONTINUUM THEORY OF ELECTROMAGNETIC FIELD & STRAIN BUBBLES

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ABSTRACT

This paper presents an entirely different, unorthodox point of view regarding the description of physical world at the ultramicroscopic level. The old concept of elastic ether medium has been revised to preclude its self contradictory properties. As per the revised concept, our familiar space-time continuum with the characteristic property of permittivity ε_0 and permeability $\mu_{0,}$ behaves as a perfect isotropic elastic continuum with elastic constant $1/\varepsilon_0$ and inertial constant μ_0 . For this 'Elastic Continuum' the equilibrium equations of elasticity are found to be identical with vector wave equation of Maxwell's electromagnetic theory. Particular solutions of these equilibrium equations as functions of space-time coordinates, satisfying appropriate boundary and stability conditions within a bounded region, are shown to represent various 'strain bubbles' and 'strain wave fields'. The electromagnetic field as well as all other forms of energy and matter are shown to exist in the Elastic Continuum as strain wave fields or strain bubbles. Through analysis of various strain bubbles, we can study the structure of various elementary or composite particles like electron, proton etc. and deduce their mutual interactions.

Keywords. Elastic Continuum; Equilibrium equations; Strain bubbles; Elementary particles.

Ether : **'It should no longer be regarded as a substance but simply as the totality of those physical quantities which are to be associated with matter free space.'**

Albert Einstein

1. INTRODUCTION

1.1 In the 19th century Physics, light waves were regarded as undulations in an all pervading elastic medium called 'ether'. The successful explanation of diffraction and interference phenomenon in terms of ether waves made the notion of the ether so familiar that its existence was taken for granted. However, in order to accommodate the notion of ether in the framework of physical universe as known at that time, some self contradicting properties had to be ascribed to this ether medium. For supporting the light waves it was required to behave like an elastic solid, but to enable the motion of material bodies through it without any resistance, it had to behave like a thin, ideal, non-viscous fluid.

1.2 Maxwell's development of the electromagnetic theory of light, rendered the ether medium superfluous as the electromagnetic field was granted an independent status, capable of independent existence and propagation in space, in accordance with Maxwell's equations. It is generally believed that the notion of ether medium was discarded as a consequence of the negative result of Michelson-Morley experiment. In fact, with the success of special theory of relativity, the Michelson-Morley experiment itself was rendered null and void. Since by the end of 19th century, the phenomenon of light waves could be explained on the basis of electromagnetic theory, there was no further necessity of retaining the concept of ether medium. Most of the elementary particles were not known to exist by then. Even the phenomena of 'matter waves' of non-electromagnetic origin, inter-convertibility of matter and energy, annihilation and materialization of particles were not known at that time. As a result the ether got discarded from the 20th century Physics so thoroughly that its non-existence is now taken for granted.

1.3 However, granting of independent status to the electromagnetic field was not sufficient by itself, we had to ascribe the characteristic properties of permittivity 'ε_0' and permeability 'μ_0' to empty space i.e. 'nothingness', which again appears self contradictory. Propagation of independent electromagnetic field through 'empty' space, at a constant velocity 'c', also depended upon the magnitude of characteristic parameters 'ε_0' and 'μ_0' ascribed to empty space. Therefore, logically it should make better sense to retain the notion of an elastic 'ether' continuum with characteristic parameters 'ε_0' and 'μ_0' ascribed to it rather than discarding 'ether' and ascribing the same characteristic parameters to 'nothingness' or empty space. If the characteristic parameters 'ε_0' and 'μ_0' are associated with an 'elastic continuum' pervading the entire space, we could view the electromagnetic waves, with energy stored in their oscillating electric and magnetic fields, as propagating through this continuum. Hence the transportation of energy across physical space could be viewed as a propagation process of specific type of waves through the elastic ether continuum. As such, for transportation of energy through the highly elastic ether continuum, it may no longer be necessary to ascribe self contradicting property of 'thin ideal fluid' to it. We may simply imagine the transportation of energy as a sort of 'propagation' process through the elastic ether continuum.

1.4 However, as a next most formidable step, it will be extremely difficult to imagine the transportation of 'matter' as a sort of 'propagation' process through the elastic ether continuum, even though the 20th century Physics has shown the equivalence and inter-convertibility between 'matter' and 'energy'. It is one thing to imagine the elementary matter particles as some sort of packets of energy entrapped in characteristic wave formation in the ether continuum, but quite another to imagine the transportation of clusters of such particles (material bodies) as a sort of 'propagation' process through the 'ether'. Yet, this most formidable step is also the most crucial one necessary to divest the self contradictory property of 'thin, ideal fluid' from the notion of highly elastic ether medium. Therefore it seems likely that all the electromagnetic phenomena, all energy entrapping and transportation processes and all wave motion that we usually believe to be occurring in empty space, are in fact occurring in the elastic 'ether' continuum with the characteristic properties of permittivity 'ε_0' and permeability 'μ_0' or elasticity constant $1/\varepsilon_0$ and inertial constant μ_0. This revised notion of ether no longer requires it to be 'thin, ideal fluid' to allow free unrestricted motion of matter through it since matter is no longer considered an independent entity separate from the 'ether'. Therefore, to distinguish this revised notion from the old ether medium of 19^{th} century, we may simply call it the 'Elastic Continuum' with associated characteristic parameters of elastic constant $1/\varepsilon_0$ and inertial constant μ_0, pervading the entire space. We may well imagine that we are just reinterpreting our familiar concept of space-time continuum with associated parameters ε_0 and μ_0, as the 'Elastic Continuum' with the associated parameters of elastic constant $1/\varepsilon_0$ and inertial constant μ_0 in appropriate units.

2. General Equations of Elasticity in the 'Elastic Continuum'

2.1 Displacement Vector Field U. Let us consider an isotropic elastic continuum pervading the entire space. Initially, let all the physical points of this continuum be represented by the corresponding geometrical points of our familiar three dimensional space referred to a suitable orthogonal coordinate system. In a conventional Cartesian coordinate system, let the x, y, z coordinates be represented by x^1, x^2, and x^3 respectively and the corresponding unit vectors **i, j, k** be represented by $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$. If O is the origin of this coordinate system, then the position vector of any point $P(x^1,x^2,x^3)$ or simply $P(x^i)$ will be given by

$$\mathbf{OP} = \mathbf{e_1}\, x^1 + \mathbf{e_2}\, x^2 + \mathbf{e_3}\, x^3 = \mathbf{e_i}\, x^i \qquad \text{(summation over i from 1 to 3)}$$

With the passage of time, physical points of the continuum may undergo certain infinitesimal displacements leading to time dependent infinitesimal deformations in the continuum. The infinitesimal displacement at any point $P(x^i)$ may be represented by a displacement vector **U** as a function of the coordinates of P as well as t .

$$\mathbf{U}(x^i, t) = \mathbf{e_1}\, u^1(x^i,t) + \mathbf{e_2}\, u^2(x^i,t) + \mathbf{e_3}\, u^3(x^i,t) = \mathbf{e_j}\, u^j(x^i,t) \qquad \ldots\ldots\ldots\ldots (1)$$

If this displacement vector **U** is finite and 'continuous' within a region of space V, then a displacement vector field $\mathbf{U}(x^i,t)$ may be said to be defined over this region of space. Specifically this displacement vector field **U**, represented by its components u^i, may be a periodic function or a combination of periodic functions of coordinates x^i and t within the field region V and may be zero at the boundaries & outside this region. Obtaining specific solutions for the displacement vector field $\mathbf{U}(x^i,t)$, under specified initial and boundary conditions, will be our major objective in the study of 'Elastic Continuum'.

2.2 Representation of Strain S. The displacement vector field $\mathbf{U}(x^i,t)$ will also represent an infinitesimal deformation field in the Elastic Continuum. The infinitesimal deformation at any point $P(x^i,t)$ is best quantified through the components of a strain tensor **S** as follows. The infinitesimal deformation or change of an arbitrary small vector $A^i(x^1,x^2,x^3,t)$ at the point $P(x^1,x^2,x^3)$ will be given by an infinitesimal affine transformation of the neighborhood of the point in question as[1],

$$\delta A^i = (\partial u^i/\partial x^j)\, A^j = u^i_{,j}\, A^j \qquad \text{(summation over } j) \qquad \ldots\ldots\ldots (2)$$

Here the quantities $u^i_{,j}$ which are the covariant derivatives of the displacement vector field u^i with respect to the coordinate x^j represent the components of strain tensor **S** such that

$$S^i_j = u^i_{,j} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

These components obviously represent only the spatial strain components. Since the displacement vector components u^i, in general will be functions of space coordinates as well as time, the partial derivatives of u^i with respect to time t (more correctly ct) that is, $(1/c).\partial u^i/\partial t$ will constitute temporal strain components. In accordance with the notions of special theory of relativity, time can be regarded as fourth dimension coordinate at right angles or in quadrature to the three space coordinates. Similarly the temporal strain component $S^i_t = (1/c).\partial u^i/\partial t$ can also be regarded as being in quadrature to corresponding spatial strain components $S^i_j = u^i_{,j}$. If however, the fourth dimensional coordinate is taken as $x^4 = \imath ct$, where $\imath$ is the complex number $\sqrt{-1}$, then corresponding to the displacement vector components u^i, the temporal strain components can be written as

$$S^i_4 = u^i_{,4} = \partial u^i/\partial x^4 = (1/\imath c).\partial u^i/\partial t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Thus, corresponding to three components of displacement vector $u^i(x^1,x^2,x^3,t)$, there will be nine spatial strain components and three temporal strain components, all of which will be functions of space and time coordinates.

2.3 In contrast to the Elastic Continuum considered above where no rigid body motion is possible, the infinitesimal deformation in elastic material media is generally split into pure deformations and rigid body motions (translations and rotations). For steady state elastic equilibrium in material media, the spatial strain components e_{ij} representing pure deformation and rotational components ω_{ij} representing rigid body motion, are given by

$$e_{ij} = (u^i_{,j} + u^j_{,i})/2 \qquad \text{and} \qquad \omega_{ij} = (u^i_{,j} - u^j_{,i})/2$$

However, in the study of the Elastic Continuum where rigid body motion is not possible, we shall not use the above mentioned e_{ij} representation for strain components. As discussed in the foregoing, we shall continue to use the total strain tensor components given by

$$S^i_j = u^i_{,j} \qquad (i \rightarrow 1 \text{ to } 3 \;\; \& \;\; j \rightarrow 1 \text{ to } 4) \qquad \ldots\ldots\ldots\ldots (5)$$

It may be quite pertinent to mention here that the displacement vector **U** and strain tensor **S** are absolute entities and are invariant under coordinate transformations. Only the magnitude of components u^i and S^i_j is dependent on the reference coordinate system and transform with coordinate transformation. Hence, the analysis of strained state of the Elastic Continuum is equally valid in all admissible coordinate systems; even though we generally prefer to use a particular coordinate system for particular problems on the overall considerations of symmetry and boundary conditions.

2.4 Representation of Stress T. At any point $P(x^1,x^2,x^3)$ of the Elastic Continuum under infinitesimal deformation, the state of stress is represented by stress tensor **T**, the components τ^i_j of which are defined as follows. With point $P(x^1,x^2,x^3)$ as the center, consider an infinitesimal plane rectangular surface area $\sigma_1 = \delta x^2.\delta x^3$, with its normal parallel to X^1- axis (Fig. 1). This infinitesimal area will have two faces. We shall consider that face of σ_1, where its unit normal ν_1 points towards positive X^1-axis, as +ve face and denote it as σ_{+1} . The other face, with normal pointing towards negative X^1-axis, will be considered -ve face and denoted as σ_{-1} . If the net force per unit area acting on σ_{+1} is termed $\mathbf{T_1}$, then it is obvious that the direction of $\mathbf{T_1}$ will not coincide with unit normal ν_1 in general, since this net force represents a resultant of three components. In fact this $\mathbf{T_1}$ vector acting on σ_{+1}, can be decomposed into its components along X^1, X^2 and X^3 coordinate directions as

$$\mathbf{T_1} = \mathbf{e_1}\tau^1{}_1 + \mathbf{e_2}\tau^2{}_1 + \mathbf{e_3}\tau^3{}_1 = \mathbf{e_i}\tau^i{}_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

With the same point $P(x^1,x^2,x^3)$ as the center, if we now consider another plane rectangular surface area $\sigma_2 = \delta x^1.\delta x^3$, with its normal parallel to X^2-axis, the net force per unit area $\mathbf{T_2}$ acting on σ_{+2} will then be given by

$$\mathbf{T_2} = \mathbf{e_1}\tau^1{}_2 + \mathbf{e_2}\tau^2{}_2 + \mathbf{e_3}\tau^3{}_2 = \mathbf{e_i}\tau^i{}_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Similarly

$$\mathbf{T_3} = \mathbf{e_1}\tau^1{}_3 + \mathbf{e_2}\tau^2{}_3 + \mathbf{e_3}\tau^3{}_3 = \mathbf{e_i}\tau^i{}_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

In general, for an infinitesimal rectangular plane area σ_{+j} perpendicular to x^j coordinate direction, the net force per unit area $\mathbf{T_j}$ acting on σ_{+j} will be given by

$$\mathbf{T_j} = \mathbf{e_1}\tau^1{}_j + \mathbf{e_2}\tau^2{}_j + \mathbf{e_3}\tau^3{}_j = \mathbf{e_i}\tau^i{}_j \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Here the quantities τ^i_j are the components of the stress tensor **T** at point $P(x^1,x^2,x^3)$. The stress components τ^i_j in general will be functions of space coordinates (x^1,x^2,x^3) of point P and time t.

2.5 The stress components τ^i_j are reckoned +ve if the corresponding components of force act in the directions of increasing x^i , when the surface normal is along increasing x^j axis. If on the other hand the surface normal is along -ve x^j axis, then positive values of components τ^i_j are associated with forces directed oppositely to the positive directions of x^i coordinate axes. Hence, for an infinitesimal volume element $\delta V = \delta x^1.\delta x^2.\delta x^3$ taken in the shape of a rectangular parallelepiped, with faces parallel to coordinate planes and point $P(x^1,x^2,x^3)$ as its center, the stress components τ^i_j will correspond to forces in opposite directions at the opposite ends of the parallelepiped.

Fig. 1. : Representation of stress components τ^i_1 on a surface element σ_{+1}

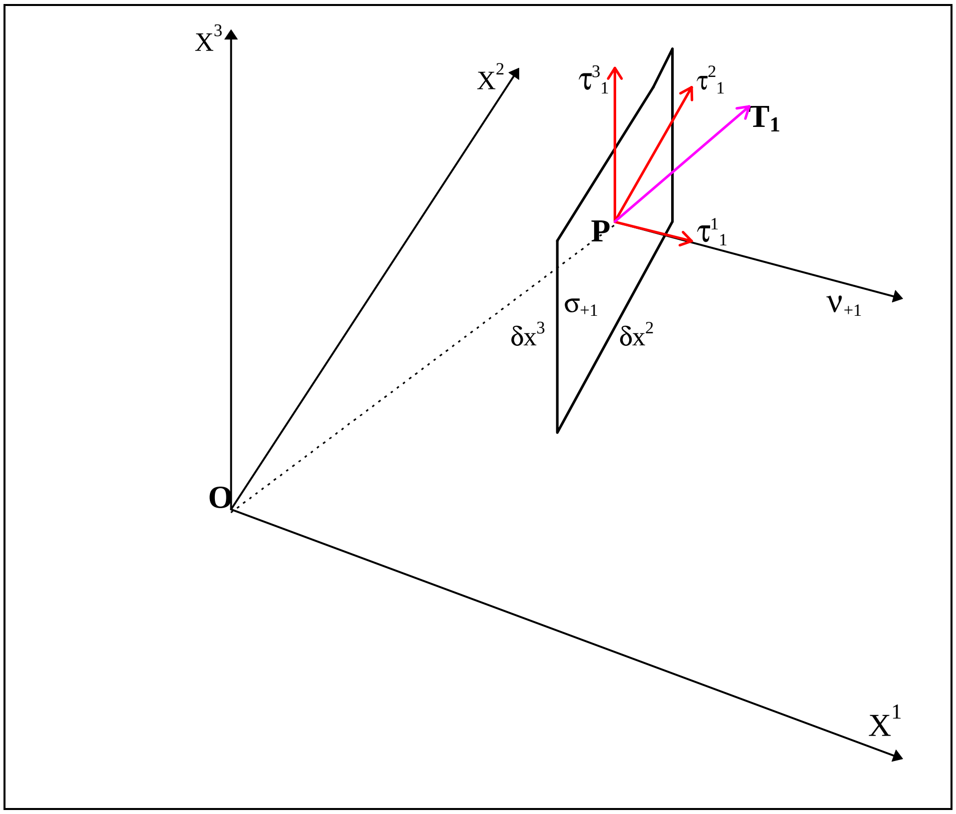
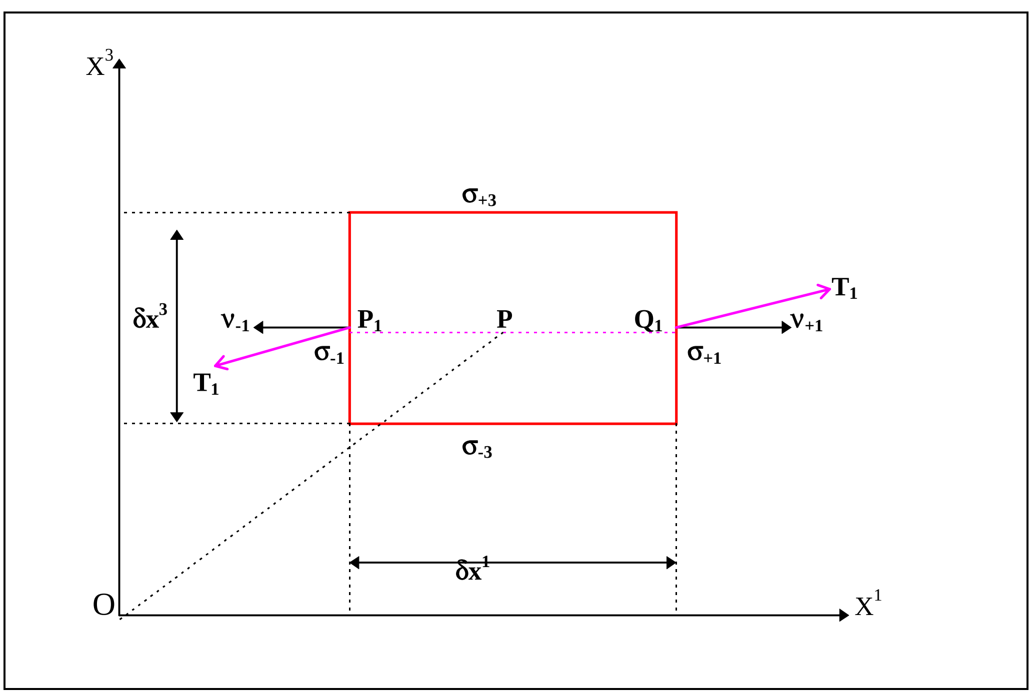


Fig. 2. : A cross section of an infinitesimal parallelepiped under stress.



2.6 <u>Dynamic Equilibrium Equations in the Elastic Continuum.</u> Ordinary material bodies, under stress, will generally be in a state of static equilibrium. However, in the Elastic Continuum, the equilibrium in a strained state is essentially dynamic. In a steady state or static equilibrium, not only the resultants of all forces acting on an infinitesimal volume element δV should vanish but the resultant moment of all forces should also vanish to ensure that pure stresses and strains do not give rise to rigid body motions and rotations. In the static equilibrium of a material body under stress, vanishing of resultant moments to avoid rigid body rotations can be ensured by the symmetry of stress and strain components τ^i_j and S^i_j . However, this condition is not applicable for the Elastic Continuum where there is neither static equilibrium nor rigid body rotations. Let us consider an infinitesimal volume element $\delta V = \delta x^1.\delta x^2.\delta x^3$ in the shape of a rectangular parallelepiped, with point $P(x^1,x^2,x^3)$ as its center and faces parallel to coordinate planes. Of this volume element, let us consider two plane faces σ_{+1} and σ_{-1} perpendicular to X^1 axis, such that point $P_1(x^1-½\delta x^1,x^2,x^3)$ is the center of σ_{-1} and point $Q_1(x^1+½\delta x^1,x^2,x^3)$ is the center of σ_{+1} (Fig. 2). Then, $P_1Q_1=\delta x^1$; $P_1P=½\delta x^1=PQ_1$ and areas of two opposite plane faces under consideration are $\sigma_{+1} = \delta x^2.\delta x^3 = \sigma_{-1}$. At any instant of time t, let us examine total forces acting on faces σ_{+1} and σ_{-1} due to the combined effect of shear and normal stresses acting on these faces. From equation (6), the total force acting on +ve face σ_{+1} is,

$$\sigma_{+1}.\mathbf{T_1}(x^1+½\delta x^1,x^2,x^3) = \delta x^2.\delta x^3[\ \mathbf{e}_1.\ \tau^1_1(x^1+½\delta x^1,x^2,x^3) + \mathbf{e}_2.\ \tau^2_1(x^1+½\delta x^1,x^2,x^3) + \mathbf{e}_3.\ \tau^3_1(x^1+½\delta x^1,x^2,x^3)]$$

Or

$$\sigma_{+1}.\mathbf{T_1}(x^1+½\delta x^1,x^2,x^3) = \delta x^2.\delta x^3[\mathbf{e}_1.\{\ \tau^1_1(x^1-½\delta x^1,x^2,x^3) + (\partial\tau^1_1(P)/\partial x^1).\delta x^1\} + \mathbf{e}_2.\{\ \tau^2_1(x^1-½\delta x^1,x^2,x^3) + (\partial\tau^2_1(P)/\partial x^1).\delta x^1\} + \mathbf{e}_3.\{\ \tau^3_1(x^1-½\delta x^1,x^2,x^3) + (\partial\tau^3_1(P)/\partial x^1).\delta x^1\}] \quad \ldots(10)$$

And the total force acting on the negative face σ_{-1} is

$$\sigma_{-1}.\mathbf{T_1}(x^1-½\delta x^1,x^2,x^3) = -\delta x^2.\delta x^3[\ \mathbf{e}_1.\ \tau^1_1(x^1-½\delta x^1,x^2,x^3) + \mathbf{e}_2.\ \tau^2_1(x^1-½\delta x^1,x^2,x^3) + \mathbf{e}_3.\ \tau^3_1(x^1-½\delta x^1,x^2,x^3)] \quad \ldots\ldots..(11)$$

2.7 Therefore, the net resultant force acting on two opposite faces σ_{+1} and σ_{-1} of the parallelepiped is obtained from equations (10) and (11) as,

$$\delta\mathbf{T_1}.\ \delta x^2.\delta x^3 = \delta x^2.\delta x^3[\mathbf{e}_1.(\partial\tau^1_1(P)/\partial x^1).\delta x^1 + \mathbf{e}_2.(\partial\tau^2_1(P)/\partial x^1).\delta x^1 + \mathbf{e}_3.(\partial\tau^3_1(P)/\partial x^1).\delta x^1]$$

Similarly considering the forces on opposite faces σ_{+2}, σ_{-2} and σ_{+3}, σ_{-3} we get the corresponding net resultant forces acting on the parallelepiped as,

$$\delta\mathbf{T_2}.\ \delta x^1.\delta x^3 = \delta x^1.\delta x^3[\mathbf{e}_1.(\partial\tau^1_2(P)/\partial x^2).\delta x^2 + \mathbf{e}_2.(\partial\tau^2_2(P)/\partial x^2).\delta x^2 + \mathbf{e}_3.(\partial\tau^3_2(P)/\partial x^2).\delta x^2]$$

and

$$\delta\mathbf{T_3}.\ \delta x^2.\delta x^1 = \delta x^2.\delta x^1[\mathbf{e}_1.(\partial\tau^1_3(P)/\partial x^3).\delta x^3 + \mathbf{e}_2.(\partial\tau^2_3(P)/\partial x^3).\delta x^3 + \mathbf{e}_3.(\partial\tau^3_3(P)/\partial x^3).\delta x^3]$$

If the body force acting on this infinitesimal volume element δV is $\mathbf{F}(x^1,x^2,x^3)$ per unit volume, then in terms of its components along coordinate directions,

$$\mathbf{F}(x^1,x^2,x^3) = \mathbf{e}_1F^1(x^1,x^2,x^3) + \mathbf{e}_2F^2(x^1,x^2,x^3) + \mathbf{e}_3F^3(x^1,x^2,x^3) \quad \text{……….} \ (12)$$

For equilibrium of the infinitesimal volume element δV, under the action of all resultant surface forces due to spatial stress components and the body forces, we have

$$\boldsymbol{\delta}\mathbf{T_1}.\ \delta x^2.\delta x^3 + \boldsymbol{\delta}\mathbf{T_2}.\ \delta x^1.\delta x^3 + \boldsymbol{\delta}\mathbf{T_3}.\ \delta x^2.\delta x^1 + \mathbf{F}\ .\ \delta x^1.\delta x^2\delta x^3 = 0 \quad \text{……….} \ (13)$$

2.8 After substituting the values of $\boldsymbol{\delta}\mathbf{T_1}$, $\boldsymbol{\delta}\mathbf{T_2}$, $\boldsymbol{\delta}\mathbf{T_3}$ and $\mathbf{F}$ from the previous equations into equation (13), we find that for the overall resultant force to vanish, its components along three coordinate directions should vanish independently. Therefore, the coefficients of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ in the above equation, after the indicated substitutions, should vanish separately. Hence,

$$\partial\tau^1{}_1(P)/\partial x^1 + \partial\tau^1{}_2(P)/\partial x^2 + \partial\tau^1{}_3(P)/\partial x^3 + F^1(x^1,x^2,x^3) = 0$$

$$\partial\tau^2{}_1(P)/\partial x^1 + \partial\tau^2{}_2(P)/\partial x^2 + \partial\tau^2{}_3(P)/\partial x^3 + F^2(x^1,x^2,x^3) = 0$$

$$\partial\tau^3{}_1(P)/\partial x^1 + \partial\tau^3{}_2(P)/\partial x^2 + \partial\tau^3{}_3(P)/\partial x^3 + F^3(x^1,x^2,x^3) = 0$$

Or in the tensor notation, the equilibrium equations reduce to a set of three partial differential equations,

$$\tau^1{}_{1,1} + \tau^1{}_{2,2} + \tau^1{}_{3,3} = \tau^1{}_{j,j} = -F^1 \quad \text{………………….} \ (14A)$$

$$\tau^2{}_{1,1} + \tau^2{}_{2,2} + \tau^2{}_{3,3} = \tau^2{}_{j,j} = -F^2 \quad \text{………………….} \ (14B)$$

$$\tau^3{}_{1,1} + \tau^3{}_{2,2} + \tau^3{}_{3,3} = \tau^3{}_{j,j} = -F^3 \quad \text{………………….} \ (14C)$$

Or simply,

$$\tau^i{}_{1,1} + \tau^i{}_{2,2} + \tau^i{}_{3,3} = \tau^i{}_{j,j} = -F^i \quad \text{…………………} \ (14)$$

Here the body force component $-F^i$ is associated with the inertial force component $\mu_0.\partial^2u^i/\partial t^2$, where $\partial^2u^i/\partial t^2$ is the acceleration corresponding to u^i and μ_0 is the inertial constant for the Elastic Continuum. Therefore, the equilibrium equation (14) may be rewritten as,

$$\tau^i{}_{1,1} + \tau^i{}_{2,2} + \tau^i{}_{3,3} = \tau^i{}_{j,j} = \mu_0.\partial^2u^i/\partial t^2 \quad \text{…………………} \ (15A)$$

And in orthogonal curvilinear coordinates with metric tensor[2] components g^{ij}, the general equilibrium equations for the Elastic Continuum take the form,

$$g^{11}\tau^i{}_{1,1} + g^{22}\tau^i{}_{2,2} + g^{33}\tau^i{}_{3,3} = g^{jj}\tau^i{}_{j,j} = \mu_0.\partial^2u^i/\partial t^2 \quad \text{………………} \ (15)$$

3. Stress - Strain Relations in the Elastic Continuum.

3.1 Modified Hooke's Law In the generalized Hooke's law for elastic material bodies, the effect of 'atomicity' or structural discreteness gets accommodated through the Poisson's ratio constant. Further, the effect of a finite value of Poisson's ratio constant for a material body is manifested through different values of speed of propagation of transverse and longitudinal strain waves. Therefore, in contrast to an elastic material body, we shall take the Poisson's ratio constant for the Elastic Continuum to be zero to ensure same speed of propagation of transverse and longitudinal strain waves.

With this, the generalized Hooke's law will get modified to a simple form as,

$$\tau^i_j = (1/\varepsilon_0).S^i_j = (1/\varepsilon_0).u^i_{,j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where $(1/\varepsilon_0)$ is the elastic constant for the Elastic Continuum, in appropriate units. In conventional electrical units the dimensions of $(1/\varepsilon_0)$ are $Nm^2/Coul^2$. However, in mechanical units the dimensions of elastic constant $(1/\varepsilon_0)$ are required to be N/m^2. Hence to ensure the compatibility of electrical and mechanical units in the Elastic Continuum, we must assign the dimension of $[M^0L^2T^0]$ or m^2 to the electrical unit Coulomb. One most tentative or rough estimate for the equivalence of Coulomb is that 1 Coulomb $\approx 10^{-22}$ m^2.

3.2 Substituting this relation (16) in the dynamic equilibrium equation (15) we get the corresponding equilibrium equation in terms of displacement components u^i as,

$$(1/\varepsilon_0).[\, g^{11}u^i_{,11} + g^{22}u^i_{,22} + g^{33}u^i_{,33}\,] = (1/\varepsilon_0).\, g^{jj}u^i_{,jj} = \mu_0.\partial^2u^i/\partial t^2 \quad \ldots\ldots\ldots..(17A)$$

Or $$g^{11}u^i_{,11} + g^{22}u^i_{,22} + g^{33}u^i_{,33} = g^{jj}u^i_{,jj} = \varepsilon_0\mu_0.\partial^2u^i/\partial t^2 = (1/c^2)\,\partial^2u^i/\partial t^2 \quad \ldots\ldots\ldots..\ (17)$$

Thus the dynamic equilibrium equation for the Elastic Continuum comes out to be the standard vector wave equation involving displacement vector components u^1, u^2 and u^3. In conventional Cartesian coordinate system (x,y,z), with physical components of the displacement vector **U** given by u^x, u^y and u^z equation (17) reduces to a set of three second order partial differential equations as,

$$\partial^2u^x/\partial x^2 + \partial^2u^x/\partial y^2 + \partial^2u^x/\partial z^2 = (1/c^2)\,\partial^2u^x/\partial t^2 \quad \ldots\ldots\ldots.(18A)$$

$$\partial^2u^y/\partial x^2 + \partial^2u^y/\partial y^2 + \partial^2u^y/\partial z^2 = (1/c^2)\,\partial^2u^y/\partial t^2 \quad \ldots\ldots\ldots.(18B)$$

$$\partial^2u^z/\partial x^2 + \partial^2u^z/\partial y^2 + \partial^2u^z/\partial z^2 = (1/c^2)\,\partial^2u^z/\partial t^2 \quad \ldots\ldots\ldots.(18C)$$

These three equations may be grouped into one equation involving vector **U** as,

$$\partial^2\mathbf{U}/\partial x^2 + \partial^2\mathbf{U}/\partial y^2 + \partial^2\mathbf{U}/\partial z^2 = \nabla^2\mathbf{U} = (1/c^2)\,\partial^2\mathbf{U}/\partial t^2 \quad \ldots\ldots\ldots..(18)$$

3.3 <u>Strain Wave Propagation in the Elastic Continuum.</u> In the above equation (18), the displacement vector **U** may be expressed as a combination of two functions; a vector function **f**(x,y,z,t) and a scalar function ψ(x,y,z,t) as,

$$\mathbf{U} = \nabla\times\mathbf{f} + \nabla\psi \quad \ldots\ldots\ldots\ldots \quad (19)$$

Here each of the functions **f** and ψ will satisfy equation (18) as,

$$\partial^2\mathbf{f}/\partial x^2 + \partial^2\mathbf{f}/\partial y^2 + \partial^2\mathbf{f}/\partial z^2 = \nabla^2\mathbf{f} = (1/c^2)\,\partial^2\mathbf{f}/\partial t^2 \quad \ldots\ldots\ldots.(20A)$$

and $$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 = \nabla^2\psi = (1/c^2)\,\partial^2\psi/\partial t^2 \quad \ldots\ldots\ldots.(20B)$$

If $\psi = 0$, then $$\nabla.\mathbf{U} = 0 \quad \ldots\ldots\ldots..\ (21)$$

The equations (18) & (20A) will therefore represent solinoidal or transverse strain wave propagation through the Elastic Continuum. If on the other hand **f** = 0, then $\nabla\times\mathbf{U}$ will also be zero and equations (18) & (20B) will represent irrotational or longitudinal strain wave propagation through the Continuum. In both of these cases, the spatial strain components as functions of space-time coordinates will be given by the terms $\partial u^x/\partial x$, $\partial u^x/\partial y$, $\partial u^x/\partial z$, $\partial u^y/\partial x$, $\partial u^y/\partial y$, $\partial u^y/\partial z$, $\partial u^z/\partial x$, $\partial u^z/\partial y$ and $\partial u^z/\partial z$, whereas the temporal strain components as functions of space and time coordinates will be given by the terms $(1/c)\partial u^x/\partial t$, $(1/c)\partial u^y/\partial t$ and $(1/c)\partial u^z/\partial t$.

3.4 Inertial Property of the Elastic Continuum. Viewing the above mentioned spatial and temporal strain components as occurring in the four dimensional space-time continuum, we recall from equation (4) and (5) that,

$$S^i_4 = u^i_{,4} = \partial u^i/\partial x^4 = (1/\imath c).\partial u^i/\partial t \qquad \text{where } x^4 = \imath ct$$

and $$S^i_j = u^i_{,j} \qquad (i \rightarrow 1 \text{ to } 3 \ \ \& \ \ j \rightarrow 1 \text{ to } 4)$$

With conventional Cartesian coordinate system (x,y,z), let the fourth coordinate x^4 be represented by η such that $x^4 = \eta = \imath ct$ and $\partial u^i/\partial \eta = (1/\imath c).\partial u^i/\partial t$. The inertial term in equation (18) will therefore change to $(1/c^2)\, \partial^2\mathbf{U}/\partial t^2 = - \partial^2\mathbf{U}/\partial \eta^2$. Accordingly the dynamic equilibrium equation (18) will transform to,

$$\partial^2\mathbf{U}/\partial x^2 + \partial^2\mathbf{U}/\partial y^2 + \partial^2\mathbf{U}/\partial z^2 + \partial^2\mathbf{U}/\partial \eta^2 = 0 \qquad \text{………..} (22)$$

This shows that in the four dimensional representation of dynamic equilibrium equations the inertial term is no longer explicit. In other words, the inertial constant μ_0 of the Elastic Continuum may be depicted in terms of its elastic constant $1/\varepsilon_0$ and velocity of light c as $\mu_0 = (1/\varepsilon_0).(1/c^2)$. As such, a finite (i.e. non-zero) value of the inertial constant μ_0 may be attributed to the finite (i.e. less than infinite) value of c in the space-time continuum. Hence the inertial property of the Elastic Continuum may be viewed as a consequence of finite value of velocity of light in the space-time continuum. In fact, even the dynamic equilibrium equation (22), in four dimensional space-time continuum may be derived ab-initio by considering the resultant surface forces on an infinitesimal four-dimensional parallelepiped, as at paras 2.6 to 3.1 above. The derivative of temporal stresses acting on 'faces' perpendicular to the fourth coordinate x^4 or η axis, will constitute the fourth term in equation (22) above, without invoking the concept of inertial body force.

4. Electromagnetic Field Equations in the 'Elastic Continuum'

4.1 In vacuum or 'free space' with characteristic permittivity ε_0 and permeability μ_0 , the electromagnetic field equations in terms of usual **E** and **B** field vectors are,

$$\nabla.\mathbf{E} = 0 \qquad \text{…………………} \quad (23A)$$

$$\nabla.\mathbf{B} = 0 \qquad \text{………………..} \quad (23B)$$

$$\nabla\times\mathbf{E} = - \partial\mathbf{B}/\partial t \qquad \text{………………..} \quad (23C)$$

$$\nabla\times\mathbf{B} = (1/c^2).\, \partial\mathbf{E}/\partial t \qquad \text{………………….} \quad (23D)$$

The transverse electromagnetic waves in 'free space', characterized by zero divergence are represented by the following standard wave equations,

$$\nabla^2\mathbf{E} = (1/c^2)\,\partial^2\mathbf{E}/\partial t^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24A)$$

$$\nabla^2\mathbf{B} = (1/c^2)\,\partial^2\mathbf{B}/\partial t^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24B)$$

Obviously, the equations (23A), (23B), and (24) above are identical in form to the equations (21) and (18), representing solinoidal or transverse strain wave propagation through the Elastic Continuum. This identity in 'form' is extended to an identity in 'essence' through the following correlation between displacement vector field **U** or the corresponding temporal and spatial strain components and the electromagnetic field vectors **E** and **B** ,

$$\mathbf{E} = -\,(1/\varepsilon_0).(1/c).\partial\mathbf{U}/\partial t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25A)$$

$$\mathbf{B} = (1/c).(1/\varepsilon_0).\,(\nabla\times\mathbf{U}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25B)$$

Through this correlation, in conjunction with equation (21), the electromagnetic field equations (23) are also satisfied identically. That means, the electric field vector **E**, in essence represents the 'temporal stress' field in the Elastic Continuum and is always a function of space and time coordinates. The magnetic field vector **B** on the other hand represents in essence (1/c) times the 'torsional stress' in the Elastic Continuum and is also a function of space and time coordinates. Therefore, we may conclude that as a logical consequence of reinterpreting space-time continuum as the 'Elastic Continuum' at para 1.4 above, the electromagnetic field in the so called 'vacuum' comes out to be a dynamic stress-strain field in the corresponding Elastic Continuum.

4.2 From equation (25A) above, it can also be seen that Maxwell's electric displacement **D** given by $\mathbf{D} = -\,(1/c).\partial\mathbf{U}/\partial t$, actually represents temporal strain component in the Elastic Continuum. One most pertinent point to be noted here is that at any given point in the continuum, the displacement vector **U** and the strain tensor **S** provide more complete information regarding the physical state of the continuum at that point than do the electromagnetic field vectors **E** and **B**.

4.3 The above mentioned stress-strain tensor concepts are mainly associated with electromagnetic field vectors defined in matter free space. The unit volt/m identified with electric field vector **E** is seen to be equivalent to Joule/Coulomb.m or Newton/Coulomb which as per the remarks at para 3.1 above, can be further reduced to N/m^2 - a unit of physical stress in the elastic continuum. However in a region of space influenced by the presence of electric charges in the vicinity, one component of electric field vector **E** is obtained as a gradient of Coulomb potential ϕ, which is essentially an interaction parameter. The Coulomb interaction potential ϕ, as will be seen later, is a consequence of or the end result of mutual interactions among various charged particles. Thus the electric field **E** obtained as a gradient of ϕ, represents an interaction force acting on mutually interacting charged particles and is strictly not the same thing as physical stress in the elastic continuum. But the equivalence of the practical units of **E** representing the physical dynamic stress and those of **E** representing mutual interaction force among charged particles, permits us to use both these concepts side by side without much distinction.

5. Equilibrium Equations in Spherical & Cylindrical Coordinates

5.1 The general equilibrium equation (17) given in tensor notation can be easily adapted to any particular coordinate system with metric tensor g^{ij} . Rewriting this equation, we have

$$g^{11}u^i_{,11} + g^{22}u^i_{,22} + g^{33}u^i_{,33} = g^{jj}u^i_{,jj} = (1/c^2)\,\partial^2 u^i/\partial t^2 \qquad \ldots\ldots\ldots\ldots\ldots\ (26)$$

where the terms $u^i_{,jj}$ represent second order covariant derivative terms of displacement vector u^i. However, for physical applications we have to finally convert all covariant and contravariant tensor components to their corresponding physical components. Some of the important steps that are relevant for adaptation of the tensor equations of elasticity to spherical polar, cylindrical or any other orthogonal coordinate system, involving physical components of displacement vector **U**, are given below

(a) The covariant derivative of u^i is given by,

$$u^i_{,j} = \partial u^i/\partial x^j + \Gamma^i_{\alpha j}\, u^\alpha \qquad \text{(summation over } \alpha) \qquad \ldots..\ (27)$$

where Γ^i_{jk} are the Christoffel symbols of second kind.

(b) The second covariant derivative of $u^i_{,j}$ is given by

$$u^i_{,jj} = \frac{\partial u^i_{,j}}{\partial x^j} + \Gamma^i_{\alpha j}\, u^\alpha_{,j} - \Gamma^\alpha_{jj}\, u^i_{,\alpha} \qquad \text{(summation over } \alpha \text{ only)} \qquad \ldots\ldots\ (28)$$

(c) Physical components of strain, which must be dimensionless, are given by

$$S^{x^i}_{x^j} = \sqrt{g_{ii}}\,.u^i_{,j}.\sqrt{g^{jj}} \qquad \text{(no summation over i or j)} \qquad \ldots\ldots\ldots..\ (29)$$

(d) The physical components of displacement vector **U**, which must have the dimensions of length [L], corresponding to the contravariant components u^i are given by

$$u^{x^i} = \sqrt{g_{ii}}\,.u^i \qquad \ldots\ldots\ldots\ldots\ldots\ldots(30)$$

(e) The physical components of temporal strain, which again must be dimensionless, corresponding to the time derivative of contravariant components u^i , are given by

$$S^{x^i}_t = \frac{1}{c}\frac{\partial u^{x^i}}{\partial t} = \sqrt{g_{ii}}\,\frac{1}{c}\frac{\partial u^i}{\partial t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(31)$$

5.2 Spherical Polar Coordinates. Let us now consider a spherical polar coordinate system given by $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$ coordinates, related to conventional Cartesian coordinates x, y, z as

$$x = r \sin\theta \cos\phi \quad ; \quad y = r \sin\theta \sin\phi \quad ; \quad z = r \cos\theta \qquad \ldots\ldots\ldots..\ (32)$$

The non-zero metric tensor components g_{ij} and g^{ij} for this coordinate system are

$g_{11} = 1$; $g_{22} = r^2$; $g_{33} = r^2 \sin^2\theta$ ………..(33A)

and $g^{11} = 1$; $g^{22} = 1/r^2$; $g^{33} = 1/(r^2 \sin^2\theta)$ ……….(33B)

The corresponding Christoffel symbols of second kind Γ^i_{jk} are given by

$\Gamma^1_{22} = - r$; $\Gamma^2_{12} = \Gamma^2_{21} = 1/r$; $\Gamma^1_{33} = - r \sin^2\theta$

and $\Gamma^2_{33} = - \sin\theta \cos\theta$; $\Gamma^3_{13} = \Gamma^3_{31} = 1/r$; $\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta$ ..……….(34)

The physical components u^r, u^θ, u^ϕ of displacement vector **U** are related to the corresponding contravariant components u^1, u^2, u^3 through equation (30) as

$u^r = u^1$; $u^\theta = r\, u^2$; $u^\phi = r \sin\theta\, u^3$ …….(35)

The physical components of spatial strain are obtained from equation (27) & (29) as

$$S^r_r = \frac{\partial u^r}{\partial r} \quad ; \quad S^r_\theta = \frac{1}{r}.\frac{\partial u^r}{\partial \theta} - \frac{u^\theta}{r} \quad ; \quad S^r_\phi = \frac{1}{r.\sin\theta}.\frac{\partial u^r}{\partial \phi} - \frac{u^\phi}{r} \qquad \text{…….(36A)}$$

$$S^\theta_r = \frac{\partial u^\theta}{\partial r} \quad ; \quad S^\theta_\theta = \frac{1}{r}.\frac{\partial u^\theta}{\partial \theta} + \frac{u^r}{r} \quad ; \quad S^\theta_\phi = \frac{1}{r.\sin\theta}.\frac{\partial u^\theta}{\partial \phi} - \frac{\cot\theta}{r}.u^\phi \qquad \text{….(36B)}$$

$$S^\phi_r = \frac{\partial u^\phi}{\partial r} \quad ; \quad S^\phi_\theta = \frac{1}{r}.\frac{\partial u^\phi}{\partial \theta} \quad ; \quad S^\phi_\phi = \frac{1}{r.\sin\theta}.\frac{\partial u^\phi}{\partial \phi} + \frac{\cot\theta}{r}.u^\theta + \frac{u^r}{r} \qquad \text{…(36C)}$$

And the corresponding physical components of temporal strain are given by

$$S^r_t = \frac{1}{c}.\frac{\partial u^r}{\partial t} \quad ; \quad S^\theta_t = \frac{1}{c}.\frac{\partial u^\theta}{\partial t} \quad ; \quad S^\phi_t = \frac{1}{c}.\frac{\partial u^\phi}{\partial t} \qquad \text{…………(36D)}$$

5.3 The dynamic equilibrium equations (17) given in tensor notation can now be rewritten in terms of physical components (u^r, u^θ, u^ϕ) of displacement vector **U**, in spherical polar coordinates, by using equations (27) to (36) above, as follows

$$\frac{\partial^2 u^r}{\partial r^2} + \frac{2}{r}.\frac{\partial u^r}{\partial r} - \frac{2}{r^2}.u^r + \frac{1}{r^2}\left(\frac{\partial^2 u^r}{\partial \theta^2} + \cot\theta.\frac{\partial u^r}{\partial \theta} + \frac{1}{\sin^2\theta}.\frac{\partial^2 u^r}{\partial \phi^2}\right) - \frac{2}{r^2}\left(\frac{\partial u^\theta}{\partial \theta} + \cot\theta.u^\theta + \frac{1}{\sin\theta}.\frac{\partial u^\phi}{\partial \phi}\right) = \frac{1}{c^2}.\frac{\partial^2 u^r}{\partial t^2}$$

$$\frac{\partial^2 u^\theta}{\partial r^2} + \frac{2}{r}.\frac{\partial u^\theta}{\partial r} - \frac{u^\theta}{r^2.\sin^2\theta} + \frac{1}{r^2}\left(\frac{\partial^2 u^\theta}{\partial \theta^2} + \cot\theta.\frac{\partial u^\theta}{\partial \theta} + \frac{1}{\sin^2\theta}.\frac{\partial^2 u^\theta}{\partial \phi^2}\right) + \frac{2}{r^2}\left(\frac{\partial u^r}{\partial \theta} - \frac{\cot\theta}{\sin\theta}.\frac{\partial u^\phi}{\partial \phi}\right) = \frac{1}{c^2}.\frac{\partial^2 u^\theta}{\partial t^2}$$

$$\frac{\partial^2 u^\phi}{\partial r^2} + \frac{2}{r}.\frac{\partial u^\phi}{\partial r} - \frac{u^\phi}{r^2.\sin^2\theta} + \frac{1}{r^2}\left(\frac{\partial^2 u^\phi}{\partial \theta^2} + \cot\theta.\frac{\partial u^\phi}{\partial \theta} + \frac{1}{\sin^2\theta}.\frac{\partial^2 u^\phi}{\partial \phi^2}\right) + \frac{2}{r^2.\sin\theta}\left(\frac{\partial u^r}{\partial \phi} + \cot\theta.\frac{\partial u^\theta}{\partial \phi}\right) = \frac{1}{c^2}.\frac{\partial^2 u^\phi}{\partial t^2}$$

…………………..(37)

Equilibrium equations (37) constitute a set of three simultaneous partial differential equations involving displacement vector components u^r, u^θ and u^ϕ. Unlike the case of equilibrium equations (18) in conventional Cartesian coordinate system, these equations in spherical polar coordinates may be considered 'mutually coupled' in the sense that none of these equations can be solved independent of one another.

5.4 Cylindrical Coordinates. In a cylindrical coordinate system defined by $x^1 = \rho$, $x^2 = \phi$ and $x^3 = z$, related to conventional Cartesian coordinates x, y, z as,

$$x = \rho \cos\phi \quad ; \quad y = \rho \sin\phi \quad ; \quad z = z \qquad \ldots\ldots\ldots\ldots\ldots (38)$$

The non-zero metric tensor components g_{ij} and g^{ij} for this coordinate system are

$$g_{11} = 1 \quad ; \quad g_{22} = \rho^2 \quad ; \quad g_{33} = 1 \qquad \ldots\ldots\ldots\ldots\ldots (39A)$$

and $$g^{11} = 1 \quad ; \quad g^{22} = 1/\rho^2 \quad ; \quad g^{33} = 1 \qquad \ldots\ldots\ldots\ldots\ldots (39B)$$

The corresponding Christoffel symbols of second kind Γ^i_{jk} are given by

$$\Gamma^1_{22} = -\rho \quad ; \quad \Gamma^2_{12} = \Gamma^2_{21} = 1/\rho \qquad \ldots\ldots\ldots\ldots\ldots (40)$$

The physical components u^ρ, u^ϕ, u^z of displacement vector **U** are related to the corresponding contravariant components u^1, u^2, u^3 through equation (30) as

$$u^\rho = u^1 \quad ; \quad u^\phi = \rho\, u^2 \quad ; \quad u^z = u^3 \qquad \ldots\ldots\ldots\ldots (41)$$

The physical components of spatial strain are obtained from equation (27) & (29) as

$$S^\rho_\rho = \frac{\partial u^\rho}{\partial \rho} \quad ; \quad S^\rho_\phi = \frac{1}{\rho}.\frac{\partial u^\rho}{\partial \phi} - \frac{u^\phi}{\rho} \quad ; \quad S^\rho_z = \frac{\partial u^\rho}{\partial z} \qquad \ldots\ldots\ldots (42A)$$

$$S^\phi_\rho = \frac{\partial u^\phi}{\partial \rho} \quad ; \quad S^\phi_\phi = \frac{1}{\rho}.\frac{\partial u^\phi}{\partial \phi} + \frac{u^\rho}{\rho} \quad ; \quad S^\phi_z = \frac{\partial u^\phi}{\partial z} \qquad \ldots\ldots\ldots (42B)$$

$$S^z_\rho = \frac{\partial u^z}{\partial \rho} \quad ; \quad S^z_\phi = \frac{1}{\rho}.\frac{\partial u^z}{\partial \phi} \quad ; \quad S^z_z = \frac{\partial u^z}{\partial z} \qquad \ldots\ldots\ldots (42C)$$

And the corresponding physical components of temporal strain are given by

$$S^\rho_t = \frac{1}{c}.\frac{\partial u^\rho}{\partial t} \quad ; \quad S^\phi_t = \frac{1}{c}.\frac{\partial u^\phi}{\partial t} \quad ; \quad S^z_t = \frac{1}{c}.\frac{\partial u^z}{\partial t} \qquad \ldots\ldots\ldots (42D)$$

The dynamic equilibrium equations (17) can now be rewritten in terms of physical components (u^ρ, u^ϕ, u^z) of displacement vector **U**, in cylindrical coordinates as follows

$$\frac{\partial^2 u^\rho}{\partial \rho^2} + \frac{1}{\rho}.\frac{\partial u^\rho}{\partial \rho} - \frac{u^\rho}{\rho^2} + \frac{1}{\rho^2}.\frac{\partial^2 u^\rho}{\partial \phi^2} + \frac{\partial^2 u^\rho}{\partial z^2} - \frac{2}{\rho^2}.\frac{\partial u^\phi}{\partial \phi} = \frac{1}{c^2}.\frac{\partial^2 u^\rho}{\partial t^2} \qquad \ldots\ldots\ldots\ldots (43A)$$

$$\frac{\partial^2 u^\phi}{\partial \rho^2} + \frac{1}{\rho}.\frac{\partial u^\phi}{\partial \rho} - \frac{u^\phi}{\rho^2} + \frac{1}{\rho^2}.\frac{\partial^2 u^\phi}{\partial \phi^2} + \frac{\partial^2 u^\phi}{\partial z^2} + \frac{2}{\rho^2}.\frac{\partial u^\rho}{\partial \phi} = \frac{1}{c^2}.\frac{\partial^2 u^\phi}{\partial t^2} \qquad \ldots\ldots\ldots\ldots (43B)$$

$$\frac{\partial^2 u^z}{\partial \rho^2} + \frac{1}{\rho}.\frac{\partial u^z}{\partial \rho} + \frac{1}{\rho^2}.\frac{\partial^2 u^z}{\partial \phi^2} + \frac{\partial^2 u^z}{\partial z^2} = \frac{1}{c^2}.\frac{\partial^2 u^z}{\partial t^2} \qquad \ldots\ldots\ldots\ldots\ldots (43C)$$

6. Strain Energy Density in the Elastic Continuum

6.1 In the deformed or stressed state of the Elastic Continuum, certain amount of strain energy will get stored in the region under stress. The strain energy density W at point P of the continuum, will obviously be a function of the intensity of strain at that point. Since the strain energy stored in any arbitrarily small volume δV of the

Continuum under stress, has to be positive, the strain energy density function W will be a positive definite form of the strain components S^i_j . Further, this strain energy density W or the energy of deformation per unit volume, has a physical meaning that is independent of the choice of coordinate system and hence is an invariant. Therefore, using the Clapeyron formula for the strain energy density for ordinary material bodies under static equilibrium, expressed in conventional Cartesian coordinate system, the spatial strain energy density for the Elastic Continuum may be given by,

$$W_s = ½\, \tau^i_j\, S^i_j = ½\, (1/\varepsilon_0)\, S^i_j\, S^i_j \qquad \text{(summation over i, j} \rightarrow 1 \text{ to } 3) \qquad \ldots\ldots\ldots (44)$$

$$= ½\, (1/\varepsilon_0)\, [\, (S^1_1)^2 + (S^2_2)^2 + (S^3_3)^2 + (S^1_2)^2 + (S^2_3)^2 + (S^3_2)^2 + (S^2_1)^2 + (S^1_3)^2 + (S^3_1)^2]$$

This formula for the strain energy density function W will also hold good in all other orthogonal curvilinear coordinate systems, provided we use physical strain components in place of S^i_j , as given by relation (29). Similarly, in a material body if the strain intensity varies with time, the kinetic energy density is given by $½\, \rho(\partial u^i/\partial t).(\partial u^i/\partial t)$. Therefore, the temporal strain energy density in the Elastic Continuum will be given by

$$W_t = \frac{\mu_0}{2}.\frac{\partial u^i}{\partial t}.\frac{\partial u^i}{\partial t} = \frac{1}{2\varepsilon_0}.\left(\frac{1}{c}\frac{\partial u^i}{\partial t}\right).\left(\frac{1}{c}\frac{\partial u^i}{\partial t}\right) \qquad \text{(summation over i} \rightarrow 1 \text{ to } 3) \quad \ldots\ldots (45)$$

$$= (1/2\varepsilon_0).S^i_t.\, S^i_t$$

Hence, the total strain energy density W within a particular strain field of the Continuum will be given by

$$W = W_s + W_t = (1/2\varepsilon_0).[\, S^i_j.\, S^i_j + S^i_t.\, S^i_t\,] \qquad \text{(summation over i, j} \rightarrow 1 \text{ to } 3) \quad \ldots\ldots (46)$$

6.2 However, the above equation (46) for the strain energy density is strictly valid only when the temporal strain components are in quadrature to the corresponding spatial strain components. That is, when the space and time coordinates are independent parameters in the strain functions and not interdependent or interlinked through some special wave functions. For example, when the solutions of equilibrium equations for displacement components u^i involve space and time coordinates as independent parameters, representing standing wave oscillations, the temporal strain components will be in quadrature to the corresponding spatial strain components. As will be seen later, this situation is encountered most frequently in the cores of all strain bubbles, where the strain energy density is computed by using equation (46). On the other hand when u^i involve functionally interlinked space and time coordinates, representing propagating phase waves, the temporal strain components in such a case may assume phase opposition to the corresponding spatial strain components.

6.3 Let us, as an example consider a particular solution of displacement components u^i that involve a propagating phase wave function of the type $\exp \imath(\kappa\, x^j \pm \kappa\, ct)$, then the corresponding spatial strain terms $\partial u^i/\partial x^j$ or $u^i_{,j}$ will be in phase opposition to the temporal strain terms $(1/c).\partial u^i/\partial t$. In such cases we may introduce a space-time phase parameter ψ given by

$$\psi = \kappa x^j + \kappa ct \qquad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (47A)$$

such that $$d\psi = \frac{\partial\psi}{\partial x^j}.dx^j + \frac{\partial\psi}{\partial t}.dt \qquad \text{(no summation over j)} \qquad \ldots\ldots\ldots \quad (47)$$

For surfaces of constant phase in the strain field, representing phase wave propagation

$d\psi = 0$ and from equation (47) ; $$\frac{\partial\psi}{\partial x^j}.dx^j = -\frac{\partial\psi}{\partial t}.dt \qquad \ldots\ldots\ldots\ldots\ldots.. \quad (48)$$

The $f(x).e^{i\psi}$ terms in u^i, where f(x) is any function of space coordinates alone, will also represent the surfaces of constant phase propagating along x^j coordinate. The effective total strain component S^i_j for such a case of propagating phase waves, where x^j and t are interlinked through ψ, will be given by,

$$S^i_j = \frac{du^i}{dx^j} = \frac{\partial u^i}{\partial x^j} + \frac{\partial u^i}{\partial t}.\frac{dt}{dx^j} = \frac{\partial u^i}{\partial x^j} + \frac{\partial u^i}{\partial t}.\left(-\frac{\partial\psi/\partial x^j}{\partial\psi/\partial t}\right) \qquad \text{(by using eqn. (48))}$$

$$= \frac{\partial u^i}{\partial x^j} - \frac{1}{c}.\frac{\partial u^i}{\partial t} = f'(x).e^{i\psi} + f(x).[\kappa\text{-}\kappa c/c].\ e^{i\psi} = f'(x).e^{i\psi} + 0 \qquad \ldots\ldots\ldots \quad (49)$$

That is, the temporal strain component gets subtracted from the corresponding spatial strain component. In other words, for the strain field consisting of phase waves of the type $u^i = f(x).e^{i\psi}$ propagating along x^j coordinate direction (+ve or -ve), the effective temporal strain for displacement component u^i will be in phase opposition to the corresponding spatial strain component u^i_j. This in effect implies that $e^{i\psi}$ type terms occurring in u^i will not contribute anything in the effective total strain. Hence for computing the total strain energy density in such cases, the $e^{i\psi}$ type terms occurring in various displacement components, may be treated as constants. The total strain energy density in phase wave fields discussed above, will therefore depend only on amplitude $f'(x)$ or more precisely, on rms value of the amplitude of such strain waves. We shall encounter such phase wave fields in the study of electrostatic field of charged particles.

7. <u>SOLUTION OF EQUILIBRIUM EQUATIONS</u>

7.1 When any region of the Elastic Continuum is subjected to some sort of deformation, a strain field may be said to have developed in that region. This strain field can be fully defined, including the strain energy stored in it, if the displacement vector **U** is completely determined as a function of space and time coordinates over the whole region of the Continuum under deformation. But the displacement vector components u^i can be completely determined from the detailed solution of the equilibrium equations (17) or (18), subject to the boundary conditions characterizing the given physical situation of the deformed Continuum. Hence the detailed study of any deformed or the stressed region of the Elastic Continuum primarily involves the detailed solution of the equilibrium equations subject to appropriate boundary conditions. Unlike ordinary linear differential equations, the general solutions of partial differential equations contain arbitrary functions which are difficult to adjust so as to satisfy the given boundary conditions.

7.2 Moreover, for different sets of boundary conditions, the given partial differential equations will yield different unique solutions. However, most of the boundary value problems involving linear partial differential equations, can be solved by the method of separation of variables. It involves a solution in a particular coordinate system, which breaks up into a product of functions each of which contains only one of the independent coordinate parameters. In a particular coordinate system, if the boundary conditions characterizing a given physical situation are such that the corresponding unique solution for u^i consists of a product of functions, each of which contains only one of the independent variables, the boundary conditions may be said to be 'symmetric' in that coordinate system. The method of separation of variables is applicable for the solution of equilibrium equations in a given coordinate system, if the boundary conditions are 'symmetric' in that coordinate system. Therefore, depending on 'symmetry' of the boundary conditions, an appropriate coordinate system will be used for solution of the equilibrium equations.

7.3 <u>General Boundary Conditions.</u> Let V be the total volume and Σ be the outer boundary surface of a particular region of the Elastic Continuum under stress. The general boundary conditions that must be satisfied by the displacement components u^i obtained from the solution of equilibrium equations, may be listed as

(a) The displacement components u^i must vanish at the boundary Σ and must remain finite and continuous within this boundary. The 'symmetry' of boundary conditions in a particular coordinate system will be governed by the shape of Σ.

(b) The strain components and the strain energy density must be finite and continuous within the boundary Σ of the region under consideration. On the boundary Σ the stress, and hence strain components may either vanish or be finite, periodic and preferably symmetric with respect to the center of the region, such that at any instant the surface integral of the stress vector over Σ must vanish.

(c) The total strain energy within the entire volume V must be finite and remain constant or time invariant in the absence of any external interaction.

(d) The amplitude of displacement vector components u^i will be proportional to the wave angular frequency $\omega = 2\pi\nu$ or its equivalent parameter $\omega/c = 2\pi/\lambda = \kappa$ which is the wave number of the strain wave oscillations occurring within the entire volume V of the Continuum under stress. This is due to the fact that whenever the amplitude of displacement vector **U** starts building up in any region of the Continuum, it will simultaneously start 'dissipating' or spreading out to its surroundings at velocity c. Therefore, higher magnitude of displacement vector amplitude will result whenever the rate of build up of **U** is high in comparison to c. However, this condition may be taken as a postulate at this stage. Since the dimension of displacement vector **U** has to be $[M^0L^1T^0]$, we shall take the integration constant for u^i as a dimensionless constant multiplied by $e\kappa$, where e is the magnitude of electron charge in Coulombs. With this we shall keep using the elastic constant $(1/\varepsilon_0)$ in the units of $Nm^2/Coul^2$.

8. STRAIN BUBBLES IN THE ELASTIC CONTINUUM.

8.1 Types of Strain Bubbles. A closed region of the Elastic Continuum with boundary surface Σ, that satisfies the above mentioned boundary conditions and contains a finite amount of energy stored in its strain field, may be called a 'Strain Bubble'. From the nature of boundary conditions and the equilibrium equations, it turns out that all valid solutions for displacement vector components u^i are functions of space-time coordinates representing various types of strain wave oscillations. That is, all 'Strain Bubbles' contain a constant finite amount of total strain energy and essentially consist of various strain wave oscillations within a specific boundary surface Σ of the Elastic Continuum. Three main distinguishing features of various types of strain bubbles are,

(a) Shape and symmetry of boundary surface Σ. The shape of the boundary surface Σ where the components u^i vanish altogether, is the most crucial boundary condition that governs the shape and to some extent the size of the strain bubble. If Σ is the surface of a right circular cylinder, the corresponding strain bubble may be called a 'Cylindrical Strain Bubble'. If Σ is a spherical surface, the strain bubble may be termed 'Spherical Strain Bubble' and corresponding to rectangular box shape of Σ the strain bubble may be referred as 'Cartesian Strain Bubble'. Therefore from the foregoing discussions about the 'symmetry' of Σ, it is obvious that cylindrical strain bubble solutions will be obtained from the equilibrium equations (43) written in cylindrical coordinates. Similarly, spherical and Cartesian strain bubble solutions will be obtained from equilibrium equations written in spherical and Cartesian coordinate systems respectively.

(b) Size of the Boundary Surface Σ. If the boundary surface Σ is located at finite distance from the center of a strain bubble, it may be termed a finite strain bubble. On the other hand if Σ extends to infinity, the strain bubble may be termed an infinite strain bubble.

(c) Type or Mode of Strain Wave Oscillations. Regarding the type of strain wave oscillations sustained within the boundary surface Σ, there may be standing wave type oscillations which can only occur along one or two coordinate directions, within a finite 'core' of any strain bubble. Or there may be propagating phase wave type oscillations along one of the coordinate directions, which can normally be sustained within an infinite 'field' of any strain bubble, with a sharp decay in amplitude. However, the total strain energy content stored even in an infinite field must remain finite and constant. In some situations, propagating phase wave type oscillations may be set up within a cylindrical ring type boundary surface Σ along ϕ coordinate direction, giving rise to 'spinning wave strain bubble'.

8.2 Strain Bubble Formation. We have seen above that if a certain finite amount of 'energy' is somehow transferred to a particular region of the Elastic Continuum a 'strain field' will develop in that 'deformed' region. The strain field within this particular region called the 'strain bubble', will be completely defined by the displacement vector components u^i obtained from the solution of equilibrium equations (17) subject to the boundary conditions characterizing the physical situation. One of the crucial conditions for the formation and stability of such strain bubbles is the time invariance or conservation of the total strain energy contained in the strain field. Although the strain components will always be functions of space & time coordinates, yet the strain energy density may or may not be time invariant. A further condition for the stability of strain bubbles is the time invariance of its strain energy density. Even with such constraints, a large number of different varieties of strain bubbles can exist or coexist within the Elastic Continuum. Further, all strain bubbles experience characteristic interactions among themselves.

8.3 Strain Bubble Interactions & Potential Energy. If the strain fields of two strain bubbles overlap in a certain region of the Elastic Continuum, the total strain components will be obtained by superposing the corresponding components of both the strain bubbles referred to a common coordinate system. Strain components can be transformed from one coordinate system to another as per the rules for transformation of mixed tensor components. For example, if we have to transform strain tensor components $\varepsilon^i_j(x)$ defined in coordinate system (x^i) to strain tensor components $S^i_j(y)$ in coordinate system (y^i) we first need the coordinate transformation relations of the type

$$y^i = f^i(x^1,x^2,x^3) \qquad \& \qquad x^i = F^i(y^1,y^2,y^3)$$

From these transformation relations we can obtain the Jacobian matrices of their partial derivatives $[\partial y^i/\partial x^j]$ and $[\partial x^i/\partial y^j]$. The required strain tensor components can now be obtained by using the relation

$$S^i_j(y) = (\partial y^i/\partial x^\alpha).\varepsilon^\alpha_\beta(x).(\partial x^\beta/\partial y^j) \qquad \text{..................................(50)}$$

Strain energy density and hence the total energy of the common field will be governed by the sum of squares of the resultant strain components. Interaction energy (E_{int}) of two such interacting strain bubbles may be defined as the difference between the total strain energy of the two strain bubbles with superposed strain fields (E_{sup}) and the sum of separate strain field energies of two bubbles (E_1 and E_2).

$$E_{int} = E_{sup} - (E_1 + E_2) \qquad \text{.............................. (51)}$$

If $S^i_j(1)$ represents the strain components of bubble 1 and $S^i_j(2)$ represents the corresponding strain components of bubble 2, referred to the same coordinate system then it can be easily seen from equations (46) & (51) that the interaction energy density W_{int} will be given by the sum of products of the corresponding strain components as,

$$W_{int}(1,2) = (1/2\varepsilon_0).\Sigma[\{ S^i_j(1) + S^i_j(2) \}^2 - \{ S^i_j(1) \}^2 - \{ S^i_j(2) \}^2]$$

$$= (1/\varepsilon_0).\ \Sigma[S^i_j(1).\ S^i_j(2)] \qquad (i \rightarrow 1 \text{ to } 3\ \&\ j \rightarrow 1 \text{ to } 4) \text{ ....(52)}$$

Similarly the interaction energy density in the common overlapped region of more than two strain bubbles can be easily shown to be the sum of interaction energies of each pair of interacting strain bubbles as,

$$W_{int}(1,2,3) = (1/\varepsilon_0).\ \Sigma[\{S^i_j(1).\ S^i_j(2)\} + \{ S^i_j(1).\ S^i_j(3)\} + \{ S^i_j(2).\ S^i_j(3)\}] \text{ ....(53)}$$

8.4 A negative interaction energy will imply the release of a portion of the total strain energy of the two interacting bubbles. The released energy will either transform into another strain bubble or wave packet, or transform into kinetic energy of motion of the interacting strain bubbles. In the extreme case of complete interaction between two strain bubbles with identical strain wave oscillations in opposite phase, the E_{sup} will reduce to zero and both strain bubbles may get annihilated with the released interaction energy transforming into one or more new strain bubbles or strain wave packets. Interaction of two or more strain bubbles with negative interaction energy may generally lead to the formation of a more stable configuration of strain bubbles or a single 'composite' bubble. When the cores of two or more interacting strain bubbles get partly overlapped the resulting interaction may be called 'core interaction' which is identical to the conventional 'strong interaction' encountered among nucleons and other elementary particles. However, when the centers of interacting strain bubbles are so far apart as to preclude the core interactions, their propagating phase wave fields, if any, may still get superposed resulting in a wave field interaction or simply the field interaction.

8.5 The interaction energy of a pair of mutually interacting strain bubbles may be identified with the conventional potential energy of one strain bubble with respect to the other. Thus in the case of a +ve potential energy, external work has to be done or energy has to be supplied to the system from outside to account for the increase in the combined or superposed strain field energy (E_{sup}). On the other hand, in the case of -ve potential energy, a portion (E_{int}) of the total strain energy of the two bubbles is released from the overlapped/common strain field, which is either transformed into the kinetic energy of the interacting strain bubbles or emitted out of the system as a new strain bubble or strain wave packet. Mutual attraction of two interacting strain bubbles can be easily attributed to their -ve interaction energy (more precisely, to the negative gradient of the interaction energy). Similarly, mutual repulsion of two interacting strain bubbles can be attributed to their +ve interaction energy. The field interactions, with negative interaction energy, between different 'pure' or 'composite' strain bubbles located quite far apart, will result in mutually 'bound' 'clusters' of strain bubbles. Formation of 'composite strain bubbles' through core interactions with negative interaction energy and development of mutually bound clusters of various strain bubbles, is a most significant phenomenon in the evolution of 'matter' within the Elastic Continuum. The conventional material particles may be viewed at ultra-microscopic scale as bound clusters of various composite and pure strain bubbles.

8.6 Strain Bubbles & Elementary Particles. At subatomic scale the primary constituents of matter, namely the electrons and nuclear particles are known to occupy an extremely small volume fraction of the order of 10^{-12} percent of the physical volume of any material body. The remaining bulk of intervening space is supposed to be empty or so called 'vacuum' with some electromagnetic fields 'existing' in this 'empty space'. These 'material particles' concentrated in such a small volume fraction of entire space are essentially characterized by their 'mass', 'charge' and interaction properties. In the parlance of strain bubbles existing in the Elastic Continuum, the clusters of pure and composite strain bubbles depicting 'elementary particles' are essentially characterized by their 'strain energy content', 'phase wave or strain wave fields' if any and their interaction properties. In principle, there could be an infinitely large number of different types of strain bubbles occurring in the Elastic Continuum, that may be correlated with equally large number of stable and unstable elementary particles. Therefore, it seems obvious that for deeper insight and more fundamental understanding of 'elementary' and 'composite' material particles, we must undertake detailed studies of corresponding 'pure' and 'composite' strain bubbles occurring, forming or transforming, interacting and decaying in the Elastic Continuum.

9. TYPICAL SOLUTIONS REPRESENTING STRAIN BUBBLES.

9.1 Cylindrical Strain Bubbles. A few examples of typical solutions of equilibrium equations (43) in cylindrical coordinates, that satisfy the required boundary conditions and represent some of the 'pure' strain bubbles, are given below.

(a) Stable Oscillating Core Strain Bubble. In accordance with the discussions of boundary conditions at para 7.2(d), one most important solution of equilibrium equations (43), that is independent or ϕ coordinate, is

$$u^{\rho} = A_1.e\kappa.\ J_1(x).\ Cos(qz).\ Cos(\kappa ct) \quad \ldots\ldots\ldots\ldots.. \ (54A)$$

$$u^{\phi} = A_1.e\kappa.\ J_1(x).\ Cos(qz).\ Sin(\kappa ct) \quad \ldots\ldots\ldots\ldots.. \ (54B)$$

$$u^{z} = 0 \quad .... \ (54C)$$

where A_1 is a dimensionless number, $x = (\kappa^2 - q^2)^{½}\rho$ and the boundary surface Σ is given by $-\pi/2 \le qz \le \pi/2$ & $0 \le x \le \alpha_1$ with $J_1(\alpha_1) = 0$ or α_1=3.832 . Here κ is the wave number of strain wave oscillations and separately determined (from Coulomb interaction model) to be equal to 1.73767×10^{15} m^{-1}. Strain energy density W_1 for this strain bubble, computed by using relations (42) and (46) works out to be

$$W_1 = \frac{A_1^2 e^2 \kappa^2}{2\varepsilon_0}\left[\left(\kappa^2 - q^2\right)\left\{\left(J_1^{'}(x)\right)^2 + \frac{J_1^2(x)}{x^2}\right\}Cos^2(qz) + J_1^2(x)\left\{\kappa^2 Cos^2(qz) + q^2 Sin^2(qz)\right\}\right]$$

Since this energy density is completely independent of time, the strain bubble represented by equations (54) is expected to be most stable and will be identified later with the nucleon core. After integrating W_1 over the whole volume, the total strain energy E_1 of this strain bubble works out to be

$$E_1 = \frac{\pi^2 A_1^2 e^2 \kappa \alpha_1^2 J_0^2(\alpha_1)}{2\varepsilon_0 \left(q/\kappa\right)\left(1 - q^2/\kappa^2\right)} \quad ....(55)$$

The above expression for E_1 is minimized for $q = \kappa/\sqrt{3}$. This strain bubble displays very strong radial as well axial interactions. At any point $P(\rho,\phi,z)$ within the strain field of this bubble, the displacement vector **U** can be 'seen' to be rotating at constant angular velocity κc and with constant magnitude. This rotational motion of displacement vector **U** may be visualized as an intrinsic 'spin' of the strain field. The strong interactions of this strain bubble will be sensitive to the direction of this intrinsic spin vector relative to 'spin' direction of the other interacting bubble.

(b) Unstable Oscillating Core Strain Bubbles. Three important solutions in this category are

$u^\rho = A_2.e\kappa.\ J_1(x).\ Cos(qz).\ Cos(\kappa ct)$ with $u^\phi = 0$ & $u^z = 0$ .... (56)

$u^\phi = A_3.e\kappa.\ J_1(x).\ Cos(qz).\ Sin(\kappa ct)$ with $u^\rho = 0$ & $u^z = 0$ .... (57)

and $u^z = A_4.e\kappa.\ J_0(x).\ Cos(qz).\ Sin(\kappa ct)$ with $u^\rho = 0$ & $u^z = 0$ .... (58)

where A_2, A_3, A_4 are dimensionless numbers, $x = (\kappa^2 - q^2)^{1/2}\rho$ and the boundary surface Σ is given by $-\pi/2 \le qz \le \pi/2$ & $0 \le x \le \alpha_1$ with $J_1(\alpha_1) = 0$. The strain energy density in these strain bubbles oscillates with time, thus rendering them unstable, even though the total strain energy remains time invariant. These strain bubbles are capable of strong interactions with other strain bubbles containing similar displacement vector components u^i. From the detailed study of their interactions, these strain bubbles are likely to be identified with the 'cores' of different mesons.

(c) Spinning Wave Strain Bubbles. Another important solution in cylindrical coordinates represents a strain wave 'spinning' or going round and round in a cylindrical ring shaped region Σ.

$u^\rho = A_m.e\kappa.\ J_m(x).\ Sin((m+1)\phi \pm \kappa ct).\ Cos(qz)$ ………….(59A)

$u^\phi = A_m.e\kappa.\ J_m(x).\ Cos((m+1)\phi \pm \kappa ct).\ Cos(qz)$ ………….(59B)

and $u^z = 0$ for $m \ge 1$; $x = (\kappa^2 - q^2)^{1/2}\rho$;

$-\pi/2 \le qz \le \pi/2$ and $\alpha_n \le x \le \alpha_{n+1}$ with $J_m(\alpha_n) = 0$

In view of the observations of para 6.3 above regarding phase wave fields, the strain energy density in this bubble is expected to be time invariant, thus rendering it a stable configuration. After detailed study of their interaction characteristics, this type of strain bubbles are likely to be used in major futuristic applications.

(d) Spiral Wave Strain Bubbles. Another almost similar solution for u^z (with $u^\rho = u^\phi = 0$) consists of a strain wave spiraling along the Z-axis. This type of strain bubble is likely to have negligible interaction with other strain bubbles and may represent certain neutrino type particles.

$u^z = \pm A_m.e\kappa.\ J_m(x).\ Cos(m\phi + qz \pm \kappa ct)$ ..………….(60)

for $m \ge 1$; $x = (\kappa^2 - q^2)^{1/2}\rho$; $-\pi/2 \le (m\phi + qz \pm \kappa ct) \le \pi/2$

and $0 \le x \le \alpha_1$ with $J_m(\alpha_1) = 0$

9.2 Spherical Strain Bubbles. In spherical polar coordinate system, that is, for spherically symmetric boundary surface Σ, a few important solutions of equilibrium equations (37) for displacement components u^r, u^θ, u^ϕ are

(a) Oscillating Core Strain Bubble. One lowest order solution of the equilibrium equations (37) is,

$u^r = A_e.e\kappa.(\pi/2x)^{½}. J_{1+½}(x).Cos(\kappa ct) = - A_e.e\kappa.G_1(x). Cos(\kappa ct)$ …….. (61A)

$u^\phi = A_e.e\kappa.(\pi/2x)^{½}. J_{1+½}(x).Sin(\theta).Sin(\kappa ct)= - A_e.e\kappa.G_1(x). Sin(\theta).Sin(\kappa ct)$ ….. (61B)

and $u^\theta = 0$;

where $G_1(x) = -(\pi/2x)^{½}. J_{1+½}(x) = [Cos(x)-Sin(x)/x]/x$;

$x = \kappa r$ and $0 \le x \le a_1$ with $J_{1+½}(a_1) = 0$ or $a_1 = 4.4934$

Strain energy density W_e for this strain bubble, computed by using relations (36) and (46) works out to be

$$W_e = \frac{A_e^2 e^2 \kappa^4}{2\varepsilon_0}.\left[\begin{array}{l}\left\{\left(G_1^{'}(x)\right)^2 + \frac{2G_1^2(x)}{x^2} + G_1^2(x).Sin^2(\theta)\right\}.Cos^2(\kappa ct) \\ + \left\{G_1^2(x) + \frac{G_1^2(x)}{x^2}.\left(Cos^2(\theta)+1\right) + \left(G_1^{'}(x)\right)^2.Sin^2(\theta)\right\}.Sin^2(\kappa ct)\end{array}\right]$$

which is not invariant with time, thus indicating instability of this strain bubble. Further, the total strain energy of this bubble, computed by integrating W_e over the whole volume, works out to be $E_e = 7.1356\ \pi A_e^2 e^2 \kappa/\varepsilon_0$. However, this oscillating core can degenerate into a lower energy state consisting of a part of this oscillating core surrounded by a radial phase wave or a strain wave field.

(b) Strain Bubble with Radial Wave Field. For this strain bubble, let

$H_1(x)= -(\pi/2x)^{½}. J_{-1-½}(x) = [Sin(x)+Cos(x)/x]/x$.

Displacement vector components for the core region are, from equation (61)

$u^r = - A_e.e\kappa.G_1(x). Cos(\kappa ct)$; $u^\phi = - A_e.e\kappa.G_1(x). Sin(\theta). Sin(\kappa ct)$; and $u^\theta = 0$

with $0 \le x \le b_1$ where $x = \kappa r$ and $J_{-1-½}(b_1) = 0$

For the wave field region $x \ge b_1$ let us consider another solution of equilibrium equations (37) consisting of a combination of $G_1(x)$ and $H_1(x)$ functions as follows,

$u^r = - A_e.e\kappa.\{G_1(x). Cos(\kappa ct)-H_1(x). Sin(\kappa ct)\} = - A_e.e\kappa.G_1(x,\psi_-)$
$\approx - (A_e.e\kappa/x) .Cos(\psi_-)$ .... (62A)

$u^\phi = - A_e.e\kappa.\{G_1(x).Sin(\kappa ct)+ H_1(x). Cos(\kappa ct)\}. Sin(\theta) = -A_e.e\kappa.H_1(x,\psi_-). Sin(\theta)$
$\approx -(A_e.e\kappa/x). Sin(\theta).Sin(\psi_-)$ .... (62B)

$u^\theta = 0$; here $\psi_- = x+\kappa ct$ $G_1(x,\psi_-) = [Cos(\psi_-)-Sin(\psi_-)/x]/x$;

and $H_1(x,\psi_-) =[Sin(\psi_-)+Cos(\psi_-)/x]/x$.

The strain energy density W_e for the core region is still the same as given above, but the total strain energy for the whole bubble is now decreased to $E_e = 5.04\pi A_e^2 e^2 \kappa/\varepsilon_0$

Reduction in this total energy and spread of a part of its strain energy into the radial phase wave or strain wave field renders the bubble an inherent stability even though the strain energy density still oscillates slightly. This strain bubble can be identified with the elementary particle electron and the radial strain wave field is expected to represent the electrostatic field of charge particles. The radial direction of propagation of phase waves in this solution distinguishes between the fields of electron and positron. Due to the considerations of para 6.3 above, the radial strain wave field of this bubble behaves like an A.C. voltage and the effective strain components in this field are given by the rms values of their peak magnitudes. At large distances $1/x^2$ terms may be neglected in comparison with $1/x$. The interaction energy of two overlapping 'strain wave' or electrostatic fields can then be computed easily to verify the Coulomb interaction law. Since the u^r and u^ϕ components here are in quadrature to each other, the intrinsic 'spin' occurs in this strain wave field also.

(c) Spinning Wave Core Strain Bubbles. Another important class of solutions of equilibrium equations (37) consists of spinning wave core type strain bubbles represented by a typical solution given below,

$$u^r = - A_{e1}.e\kappa.G_1(x).\ \mathrm{Sin}(\theta)\mathrm{Cos}(\theta).\mathrm{Cos}(\phi \pm \kappa ct). \qquad \text{........................(63A)}$$

$$u^\theta = A_{e1}.e\kappa.G_1(x).\ \mathrm{Sin}^2(\theta).\mathrm{Cos}(\phi \pm \kappa ct). \qquad \text{........................(63B)}$$

$$u^\phi = 0\ ;$$

where $x = \kappa\, r$ and $0 \le x \le a_1$ with $J_{1+½}(a_1) = 0$ or $a_1 = 4.4934$

This strain bubble too is expected to be inherently stable and after studying its interaction characteristics, may be identified with some neutrino type particle.

10 Kinetic Energy of Strain Bubbles and Quantum Mechanics.

10.1 Total strain energy stored in any strain bubble at 'rest' in the Elastic Continuum, may be treated as its rest mass energy or 'bound energy'. Apart from the change in their total 'bound energy' during interaction of two strain bubbles, the magnitude of dynamic stresses in their common region may either increase (positive interaction energy) or decrease (negative interaction energy) thereby disturbing the symmetric distribution of dynamic stresses in both strain bubbles. As a result of this asymmetry induced in dynamic stress field during interaction, equal and opposite resultant forces F_{int} will start acting on both strain bubbles tending to move them in such a way as to reduce their total bound energy. The motion of interacting strain bubbles may be visualized as the motion of their respective 'center of mass' points referred to a common coordinate system. With the motion of each strain bubble possessing non-zero rest mass, we associate the terms kinetic energy and momentum as per their conventional definitions. As mentioned earlier the negative interaction energy of interacting strain bubbles is the amount of energy released from their 'bound' or 'mass' energies during interaction and gets transferred to the kinetic energies of their motion in accordance with the laws of conservation of energy & momentum. The exact mechanism of transfer of interaction energy to the kinetic energy is expected to be quite a complex phenomenon and needs to be investigated separately.

10.2 The most pertinent point here is that just as all other forms of energy exist in the Elastic Continuum as strain energy of various strain bubbles, the kinetic energy associated with the motion of any strain bubble also must be existing in some sort of 'strain wave field' associated with the motion of that strain bubble. But we know from Quantum Mechanics that the only waves of non-electromagnetic origin, associated with the motion of microscopic particles, are the de Broglie waves represented by 'ψ' wave function. Hence, logically the strain wave field associated with the motion of a strain bubble, must be identified with the 'ψ wave field' associated with the motion of that strain bubble. However, as noted at para 3.3 above, the only waves of non-electromagnetic origin that could be induced in the Elastic Continuum, are the longitudinal strain waves that must therefore be identified with the 'ψ wave field'. Now we may visualize the uniform motion of a strain bubble as a state in which a moving 'ψ wave field' carrying a definite amount of total strain energy (i.e. kinetic energy), is induced or associated with the strain bubble in motion, as a consequence of its interaction with other strain bubbles. Therefore, change in motion of the strain bubble may be visualized as a process or phenomenon during which the interaction energy gets transferred to the kinetic energy or 'total strain energy of the associated ψ wave field' and vice versa . Since the bubble interactions and such energy transfer processes are limited by finite velocity of light 'c' due to their inherent 'spatial spread', classical mechanics may be considered adequate for describing the motion of strain bubbles at low velocities. However, at higher velocities and corresponding high energy interactions, adequate study and analysis of the associated phenomenon can only be made by using the techniques of special theory of relativity and Wave Mechanics. But the fundamental concepts of Wave Mechanics may have to be thoroughly revised and refined in the light of Elastic Continuum Theory.

10.3 One most important point that needs to be critically examined at this stage is the inertia property of mass or mass equivalent of energy. Logically, the energy density W in a strain bubble divided by c^2 should display the property of inertia during the motion of that strain bubble. Dimensionally too, W/c^2 may be considered equivalent to the inertial constant μ_0 for the Elastic Continuum as used in equations (15) and (17A). Therefore, it seems quite natural to extend the equilibrium equations (17A) by replacing μ_0 with $(\mu_0 + W/c^2)$ to obtain the equilibrium equations for a strain bubble in motion. Even though such extended equilibrium equations turn out to be non-linear partial differential equations in displacement vector components u^i, yet they may be indispensable for the study of longitudinal strain wave field associated with the motion of strain bubbles. Perhaps the study of such extended equilibrium equations might also provide the basis or foundations of Wave Mechanics.

11. SUMMARY AND CONCLUSION

11.1 Beginning with an axiomatic observation that our familiar space-time continuum with the characteristic property of permittivity ε_0 and permeability $\mu_{0,}$ behaves as a perfect isotropic elastic continuum with elastic constant $1/\varepsilon_0$ and inertial constant μ_0, we have given detailed description of displacement vector **U**, strain tensor **S** and stress tensor **T** in this continuum. Precluding atomicity and rigid body motions in the Elastic Continuum, we have used a simple modified form of Hooke's law and derived ab-initio the dynamic equilibrium equations of elasticity. These equilibrium equations are found to be identical with the vector wave equation of Maxwell's electromagnetic theory. Particular solutions of these equilibrium equations, as functions of space-time coordinates satisfying appropriate boundary and stability conditions within a bounded region, are shown to represent various 'strain bubbles' and 'strain wave fields'. The electromagnetic field as well as all other forms of particles, are shown to exist in the Elastic Continuum as strain wave fields or strain bubbles with definite amount of strain energy associated with them. Mutual interactions among various strain bubbles and fields are shown to be governed by the increase or decrease in strain intensity in their common superposed strain field. The clusters of pure and composite strain bubbles depicting 'material particles' are essentially characterized by their 'strain energy content', 'phase wave or strain wave fields' if any and their interaction properties. Therefore, it is imperative that for deeper insight and more fundamental understanding of 'elementary' and 'composite' material particles and the associated phenomenon at ultra microscopic level, we must undertake detailed studies of corresponding strain bubbles occurring, transforming, moving, interacting and decaying in the Elastic Continuum.

References

[1] I. S. Sokolnikoff, Mathematical Theory of Elasticity, McGraw-Hill Book Co. Second Edition, (New York - 1956) Chapter 1.

[2] I. S. Sokolnikoff, Tensor Analysis, John Wiley & Sons, Inc. Second Edition , (New York - 1964) Chapter 2.