R E S O N A N C E   A N D   M A X I M U M   P O W E R   T R A N S F E R

By Luiz Amaral
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Discussions about this subject are very common and it seems that many technicians still have some difficulty to understand it correctly.

Let's suppose a system under sinusoidal AC. The question is: is the load resonance necessary to occur the maximum power transfer to it from a generator?

The answer is: no.

Let's observe the circuit of Figure 1:

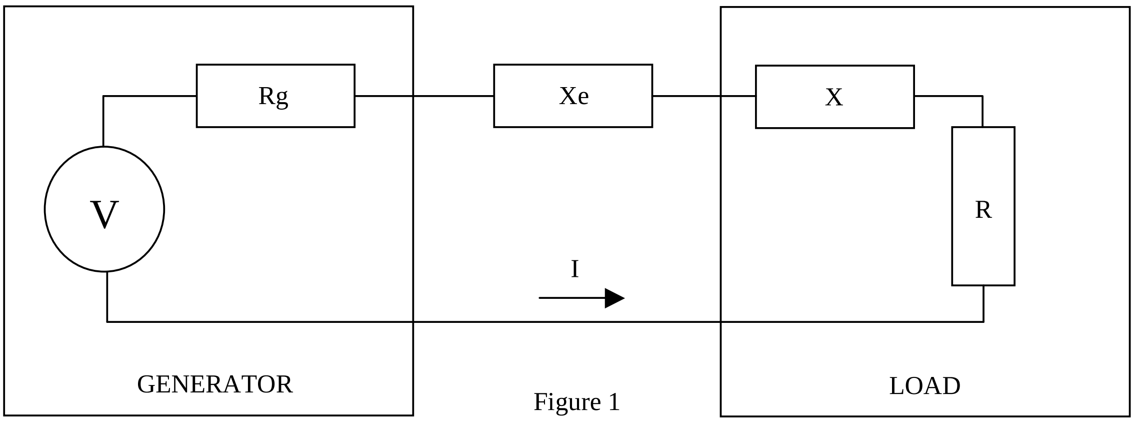

Figure 1

A generator with voltage **V** and internal resistance **Rg** (we suppose, as usual, that the generator has no internal reactance; this doesn't alter the derivation generality) is coupled to a load with resistive component **R** and relative component **X**, through an auxiliary external reactance **Xe**.

Which is the condition for maximum power transfer from the generator to the load with constant components, that is, the circuit is linear and time independent?

We must remember that, here, the generator is given, therefore **V** as **Rg** are fixed, we having the liberty to vary **R**, **X** and **Xe** to get such maximum power transfer.

We know that, with no load reactance, the maximum power is transferred to the load (now a pure resistance **R**) when the latter is equal to the generator resistance **Rg**. So, let’s make **R=Rg**.

Let’s calculate the current **I** in that condition of equal resistances:

$I=V/Z$ (1), where **Z** is the total impedance seen by the generator.

But **$Z=2.Rg+j.(Xe+X)$ (2)**

With its module given by:

$|Z|=\sqrt{[4.Rg^2+(Xe+X)^2]}$ (3)

In **(1)** we see that **I** is a maximum when **Z** is minimum and, so, we search for a minimum in **(2)**:

Here the only variable is **Xe+X** that, being squared, has its minimum when it is zero. This is possible because reactances may be negative. Thus, **Z** will be a minimum when **Xe=-X**, that is, the reactances have the same value, but opposite signals.

This shows that, if **X is** capacitive, **Xe** must be ser inductive and vice-versa, that is, for maximum power transfer, we must have resonance, but ***of the global circuit*** and ***not*** of the load itself, as **Xe** is external to it (verify again the question made in the beginning of this article).

As a classic example we have a vertical antenna fed against the ground and less than ¼ wavelength long, therefore, a non-resonant radiator and presenting a capacitive reactance, as in Figure 2, where the resistive component is the radiation resistance.

How to couple it to a transmitter in order to get maximum power transfer?

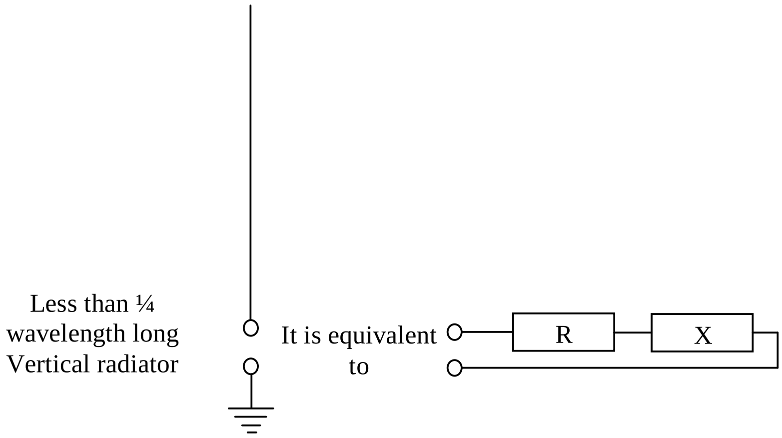

Figure 2

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There are two equivalent methods as to results, but conceptually slight different. The first consists simply in putting an inductor ***in the radiator*** and, therefore, being constituent of it, as to get the total reactance cancellation, as in Figure 3.

The second method, as showed in Figure 4, consists in putting the inductive reactance in the signal path from the transmitter to the antenna and close to the latter. The rectangles involving the radiators in both figures call the attention to the fact that, in the first case, the radiator was made resonant, but in the second one, it was not. In both cases the current on the radiator was maximized, promoting the maximum power transfer, that is, the current ***path*** impedance was minimized.

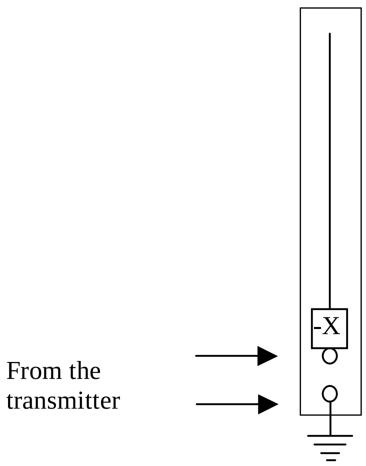

Figure 3

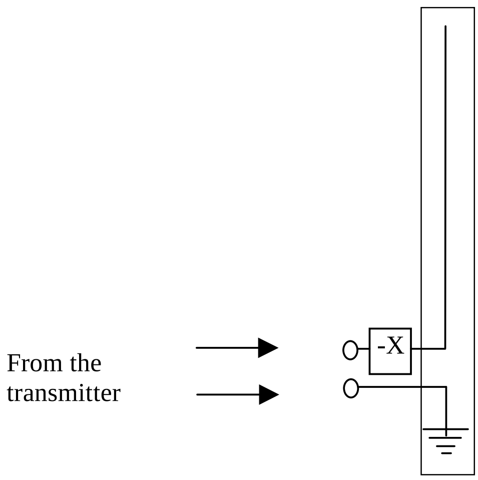

Figure 4

We can conclude, then, that for maximum power transfer from a generator to a load, ***this*** doesn't need to be resonant, but only the ***global*** circuit, which guarantees the maximum current on the resistive component of the load.

If we consider the auxiliary reactance (that performs the resonance) as part of the load or not, is a simple question of point of view, as we may also consider it as part of the generator. In this case, we have a not purely resistive generator transferring maximum power to a non-resonant load (our radiator in the example). That is the common case of a use a coupler close to the transmitter to get the maximum power transfer to a transmission line that may present a reactive component.

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