T R A N S M I S S I O N L I N E S I N P A R A L L E L

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This article shows the result when we put in parallel two ideal transmission lines with any surge impedances and with any type of load, but with the same electrical length, that is, lengths taking into account the velocity factors of the lines.

A generator with voltage **V** is connected to two transmission lines with the same electrical lengths in parallel, that is, loaded by one common impedance **Zc**, as in Figure 1.

The surge impedances of the lines are **Zo1** and **Zo2**. The generator ‘sees’ a reflected impedance **Zr**.

On the load, the voltage is **Vc**.

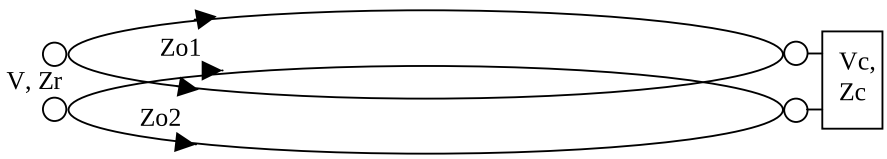

Figure 1

To analyse what happens, we separate the system into two loads **Zc1** and **Zc2**, one for each line, so that this configuration results the same impedance **Zr** to the generator.

We also want that, with the lines put in parallel, we get the circuit of Figure 1. In other words, we create a new circuit in such a way that it doesn’t matter if the loads **Zc1** and **Zc2** are paralleled or not under the generator point of view. The system with separated loads is showed in Figure 2.

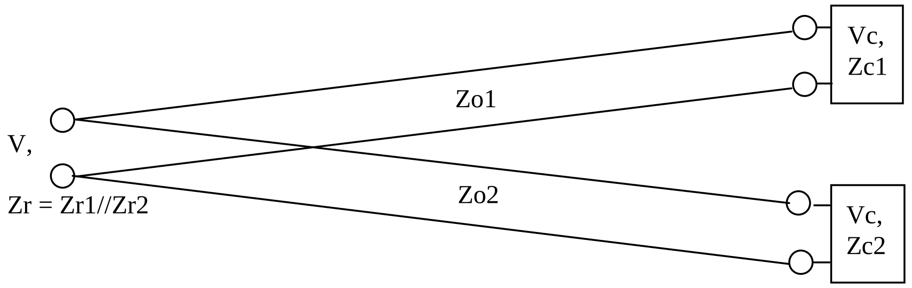

Figure 2

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In this scheme, the load **Zc1** reflects **Zr1** to the generator through the line with surge impedance **Zo1** and **Zc2** reflects **Zr2** to the generator through the line with impedance **Zo2**. **Zr1** in parallel with **Zr2** is equal to the original reflected impedance **Zr**.

As we want that the parallel of **Zc1** and **Zc2** is equal to the original load **Zc**, the phases of **Zc1** and **Zc2** must be equal, that means, both have the same reactance/resistance ratio.

Besides, as explicit in Figure 2, we need that the voltages on the loads are equal to **Vc** in amplitude and phase, as it is the original voltage on the load **Zc**. This guarantees that the two loads **Zc1** and **Zc2** can be put in parallel not affecting the generator.

So, we may write:

Zc = Zc1 . Zc2 / (Zc1 + Zc2)   [1]

If the voltages on the loads are equal, these loads are proportional to the corresponding surge impedances. So:

Zc1 / Zo1 = Zc2 / Zo2 ou Zc2 = Zo2 . Zo1 / Zc1   [2]

But **Zc = Zc1 // Zc2**:

Zc = Zc1 . Zc2 / (Zc1 + Zc2)   [3]

Applying **[2]** in **[3]**, we get:

Zc = Zc1 .Zo2 / (Zo2 + Zo1)   [4]

or **Zc1 = Zc . (Zo2 + Zo1) / Zo2   [5]**

the same for **Zc2**:

Zc2 = Zc . (Zo2 + Zo1) / Zo1   [6]

For ideal lines, the reflected impedance **zr**, for given surge impedance **zo** and load **zc**, is given by:

zr = zo . (zc + zo . t) / (zo + zc . t), onde t = j . tg β . L

Here **j** is the imaginary unity and **β = 2 . π / λ**, with **λ** being the wavelength in the line, that is, taking into account the velocity factor of the line.

Applying the last expression for lines 1 and 2, we have:

Zr1 = Zo1 . (Zc1 + Zo1 . t) / (Zo1 + Zc1 . t)   [7]

Zr2 = Zo2 . (Zc2 + Zo2 . t) / (Zo2 + Zc2 . t)   [8]

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Replacing in **[7]** e **[8]** **Zc1** e **Zc2** by their values of **[5]** and **[6]**, we have:

$$Z1r = Zo1 \cdot [Zc + t \cdot Z1o \cdot Z2o / (Z1o + Z2o)] / [Z1o \cdot Z2o / (Z1o + Z2o) + Zc \cdot t] \quad [9]$$

$$Z2r = Zo2 \cdot [Zc + t \cdot Z1o \cdot Z2o / (Z1o + Z2o)] / [Z1o \cdot Z2o / (Z1o + Z2o) + Zc \cdot t] \quad [10]$$

Let’s write:

$$P = [Zc + t \cdot Z1o \cdot Z2o / (Z1o + Z2o)] / [Z1o \cdot Z2o / (Z1o + Z2o) + Zc \cdot t] \quad [11]$$

So:

$$Z1r = Zo1 \cdot P \quad [12]$$

$$Z2r = Zo2 \cdot P \quad [13]$$

As we want that **Z1r** in parallel with **Z2r** results in **Zr**, we have:

$$Zr = Z1r \cdot Z2r / (Z1r + Z2r) \quad [14]$$

Puting **[12]** e **[13]** em **[14]**, we get:

$$Zr = Zo1 \cdot Zo2 \cdot P / (Zo1 + Zo2) \quad [15]$$

Now we want to know which is the surge impedance of the line that, alone, replaces the paralleled line pair, resulting in the same impedance **Zr** with the load **Zc**. By using the general expression for the reflected impedance, we have:

$$Zr = Zo \cdot (Zc + Zo \cdot t) / (Zo + Zc \cdot t) \quad [16]$$

Repalcing **P** in **[15]** by its definition in **[11]**, we get:

$$Zr = [Z1o \cdot Z2o / (Z1o + Z2o)] \cdot \{Zc + [Z1o \cdot Z2o / (Z1o + Z2o)] \cdot t\} / \{[Z1o \cdot Z2o / (Z1o + Z2o)] + Zc \cdot t\} \quad [17]$$

The expression **[15]** corresponds to a unique line and **[17]** to the case of two paralleled lines

Comparing both, we see that:

$$Zo = Zo1 \cdot Zo2 / (Zo1 + Zo2) \quad [18]$$

[18] shows that the surge impedance **Zo** of the line that repalaces the pair of lines in is just the parellel of the surge impedances **Zo1** e **Zo2**.

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We can say finally:

“If a generator is connected to a load through a pair of ideal transmission lines in parallel with surge impedances **Zo1** and **Zo2**, both with the same electrical length, the unique ideal line with the same electrical length that replaces that system has its surge impedance **Zo** equal to the perallel of **Zo1** and **Zo2**.”

Example:

We want a piece of cable that transforma the 25 Ohm input impedance of an antenna into 50 Ohm to use any length of 50 Ohm cable to the transmitter.

An ¼ wavelength line would do the job and its surge impedance would be given by:

$\mathbf{Zo^2 = 25 . 50\ Ohm}$, ou seja, $\mathbf{Zo \approx 36\ Ohm}$.

But this cable doesn’t exist commercially, but two pieces of a 73 Ohm cable in parallel would result in a simulated cable with 36 Ohm and would solve the problem.

In this example we have perfect impedance match and the load is pure resistive, that is, real. In the general studied case it is not necessary, the load and the electrical length may be any.

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