1/8 WAVELENGTH TRANSMISSION LINE AND CONSEQUENCES

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The expression relating the reflected impedance Z2 at one end of an ideal transmission line with surge impedance Zo loaded at the other end with a load impedance Z1 is given by:

Z2 = Z0 . (Z1 + j . t . Z0) / (Z0 + j . t . Z1) (1)

With j being the imaginary unit and t = tg (2 . π. L / λ), where λ = wavelength on the line, L = line length and π = 3,14...

Lets buid a table of t for the several values of L / λ:

L/ λ	**t**
1/8	**1**
1/4	**∞**
1/2	**0**

Z2 is a complex number, with its real part (resistance) Z2r and its imaginary part (reactance) Z2i.

Lets have Z1 real.

Rewriting **(1)** with its real and imaginary parts explicitly, we have:

$Z2 = Z0^2 . Z1 . (1 + t^2) / (Z0^2 + t^2 . Z1^2) + j . t . (Z0^2 - Z1^2) / (Z0^2 + t^2 . Z1^2)$ (2)

In **(2)** we have:

$Z2r = Z0^2 . Z1 . (1 + t^2) / (Z0^2 + t^2 . Z1^2)$ (3)

$Z2i = t . (Z0^2 - Z1^2) / (Z0^2 + t^2 . Z1^2)$ (4)

Its module is given by[1]**:**

$|Z2| = \sqrt{(Z2r^2 + Z2i^2)}$ (5)

and its phase angle by:

$tg\ \varphi = Z2i^2 / Z2r^2$ (6)

For L / λ = 1/8, the expressions **(3)** and **(4)** are:

$Z2r = 2 . Z0^2 . Z1 / (Z0^2 + Z1^2)$ (7)

$Z2i = (Z0^2 - Z1^2) / (Z0^2 + Z1^2)$ (8)

Its module:

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[1] **We must remember that, for ideal lines, Zo is real.**

$|Z2| = Z0$ (9)

Its phase:

$tg\ \varphi = (Z0^2 - Z1^2) / (2 . Z0 . Z1)$ (10)

The interesting detail of 1/8 wavelength lines is that its module is constant, independent of the resistive load and equal to Zo[2]**.**

So, if we load the 1/8 wavelength with a variable resistor R, for example, we will get a Z2 with module Z0, independent of R and (in the expression (10) if we make Z1 – R) its phase varying from $+\pi/2$, for R = 0, passing through 0, for R = Z0, up to $-\pi/2$,for $R \rightarrow \infty$.

We may use the equation (1) or (2) for calculating Z2 in the case of ¼ wavelength, with $t \rightarrow \infty$. But, instead, we can associate two pieces of 1/8 wavelength lines forming a ¼ wavelength line, using for two times the equations (7) e (8), with the reflected impedance of the first piece as load impedance of the second one (the interested reader can do it as an exercise). The result, as we could expect, is:

$Z2 = Z0^2 / Z1$ (11)

The surge impedance is the geometric average between the load and reflected impedances for a ¼ wavelength line.

In the ½ wavelength case, we may associate two ¼ wavelength lines. The reflected impedance of the first piece is given by equation (11). If we put Z2 as the load impedance for the second piece, that mean, as Z1 in equation (11), we get Z3, the reflected impedance:

$Z3 = Z0^2 / Z2 = Z0^2 / (Z0^2 / Z1)$

or **$Z3 = Z1$ (12)**

The ½ wavelength line repeats, as reflected impedance, the load one, as we already expected.

As any ½ wavelength line repeats its load impedance, any multiple of ½ wavelength line will operate according to equation (12).

If we add a multiple ¼ wavelength line (that is an even multiple of 1/8 wavelength) to a ¼ wavelength line, we get an odd number of ¼ wavelength and the equation (11) will repeat. So, for an odd multiple of ¼ wavelength, the equation (11) still holds.

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[2] **This could be concluded directly from the expression (1), with t=1.**

Similar reasoning can be used for the 1/8 wavelength case.

We may get the same results rebuilding the t values table for odd and integer multiples of 1/8, ¼ and ½ wavelengths.

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