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DOUBLE MAPPING OF ISOPARAMETRIC MESH GENERATION

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Abstract—Although coordinate-transformation techniques, such as conventional curvilinear isoparametric mesh generating method, normally produce well conditioned meshes, they are commonly the most restrictive mesh generators. This is due mainly to the fact that the number of nodes on the opposite sides of the superelement should be equal [H. Kardestuncer, *Finite Element Handbook*. McGraw-Hill, New York (1987)]. In this paper, the isoparametric method is improved for two- and three-dimensional problems, such that the opposite sides can have different number of nodes. The extended isoparametric method gives much more flexibility in choosing the number of nodes and looks promising for applying different methods of adaptive mesh generation [M. H. Kadiver and A. Korminezhad, Automatic mesh generation by triangulation, double, isoparametric mapping and bisection mapping, *Proc. 10th IASTED Int Conf.* Applied Informatic, Innsbruck, Austria (1992)]. A similar method also can be applied in a blending-function mesh generator.

1. INTRODUCTION

In the curvilinear isoparametric transformation, the number of nodes on the opposite sides of each superelements should be equal. In most of the problems, especially in three-dimensional cases, the sides of some objects, have a smaller length with respect to the other sides. Therefore, when the nodes are equal on opposite sides, the elements would be small on the smaller side and would be large on the larger one. This is one of the weak points of isoparametric transformation which causes less flexibility. The larger number of superelements means more labor work and more computer time. This restriction has made the curvilinear isoparametric transformation method an uncommon one. Therefore, it has not been used in new automatic or adaptive mesh generations. If we are able to use different numbers of nodes on opposite sides, than the above difficulties would be solved [3] and this old, but conventional mesh generation, can be re-used in automatic mesh generations [2].

1.1. *General concepts and definitions*

Since Zienkiewicz and Phillips [4] have described the theory and techniques employed in a mesh generator based on isoparametric mapping concepts, only essential points will be reiterated herein. A typical three-dimensional superelement is shown in (Fig. 1a). The X–Y–Z coordinates of a typical point within the superelement is related to the 20 pairs of nodal coordinates by

$$X = \sum_{1}^{20} N_i(\xi, \eta, \zeta) X_1$$

$$Y = \sum_{1}^{20} N_i(\xi, \eta, \zeta) Y_1$$

$$Z = \sum_{1}^{20} N_i(\xi, \eta, \zeta) Z_1, \qquad (1)$$

where N_i is the shape function associated with node i [5]. Thus, if the nodal coordinates (X_i, Y_i, Z_i) are known, the Cartesian coordinates of any specified point, ξ, η and ζ can be easily calculated from eqn (1).

This procedure is shown in Fig. 2 for two-dimensional problems. As can be seen, the number of nodes on side *bc* and *ad* should be equal, though side *bc* is smaller than side *ad*. Therefore, the size of the elements cannot be uniform (Fig. 2c). By conventional isoparametric mapping, for uniformity of the size of elements, one should divide the above superelement into smaller ones which takes a lot of computer time, and sometimes it is impossible. If the number of nodes on side *bc* is reduced, which means that opposite sides do not have equal number of nodes, an almost uniform size of elements can be obtained. For this purpose, a new method is proposed and discussed for two- and three-dimensional problems.

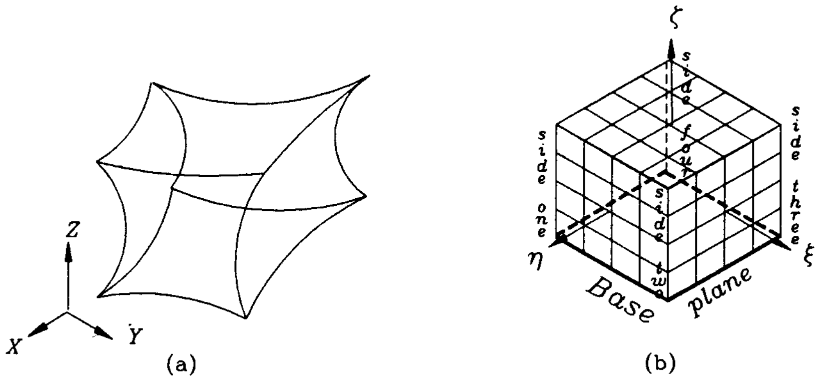


Fig. 1. General superelement in the three-dimensional coordinate; (a) a general superelement in the X–Y–Z coordinate; (b) discretized superelement in the ξ–η–ζ coordinate.

2. TWO-DIMENSIONAL PROBLEMS

In two-dimensional problems, two different cases are considered. The first, when the number of nodes on the opposite sides are different from each other, and second, when the number of nodes on all four sides are different. We take a triangular superelement as a special case of a quadrilateral one where its third and the fourth sides are coincided.

2.1. *Two-dimensional superelement with different numbers of nodes on two opposite sides*

For a superelement with different numbers of nodes on two opposite sides (Fig. 3a), by curvilinear isoparametric mapping, the superelement is transferred into a square in the ξ–η coordinate (Fig. 3b). In this coordinate, by dividing each sides into equal segments corresponding to the number of division of that side, the position of the nodes on the sides is assigned. These nodes are called the primary nodes. Since the number of nodes are not equal on two opposite sides, the superelement cannot be discretized in the ξ–η coordinate. Therefore, the side with smaller number of nodes is extended from both sides, such that the number of nodes on the two opposite sides become equal to each other (Fig. 3b). The new quadrilateral, which has the same number of nodes on two opposite sides, is transferred to the ξ'–η' coordinate (Fig. 3c), and in the new coordinate, it is discretized as usual (Fig. 3d). This discretized square in the ξ'–η' coordinate is transferred back to the ξ–η coordinate. In the ξ–η coordinate (Fig. 3b), the added part should be canceled out by transferring all the nodes which are outside of the original square to the nodes which are on the sides of the square (Fig. 3c). It is clear that for compatibility between the elements, these nodes should be transferred to the corresponding primary nodes. After transformation, the new discretized square in the ξ–η coordinate is ready for it's final transformation to the Cartesian coordinate. In the final transformation (Fig. 3d), the elimination of the extra parts in the ξ–η coordinate, may create elements with very large aspect ratio. Then, the special refinement should be used to modify the aspect ratio. This will be discussed in the next sections.

For elements with a smaller number of nodes, one can extend it in one direction, instead of two directions. This will cause distortion in the elements. In other words, all elements will be inclined into that direction which generally is not appropriate.

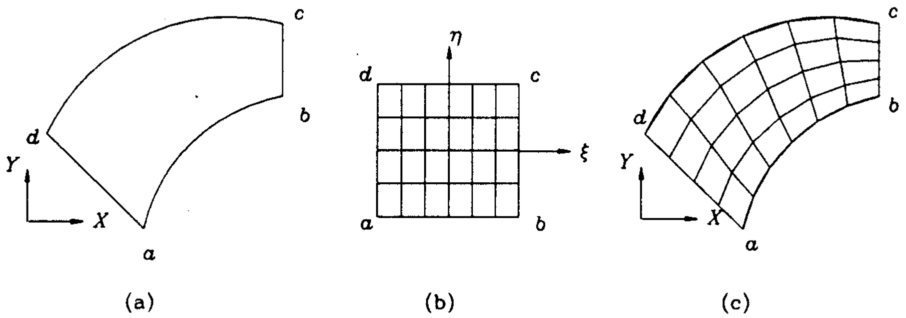


Fig. 2. General superelement in the two-dimensional coordinate; (a) a general superelement in the X–Y coordinate; (b) discretized superelement in the ξ–η coordinate; (c) discretized superelement in the X–Y coordinate.

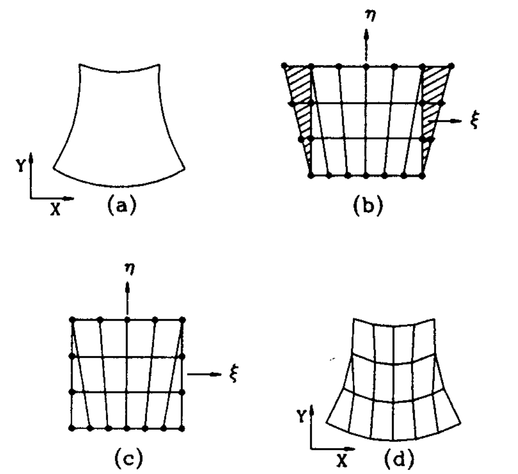


Fig. 3. Procedure of discretization when only two parts have a different number of nodes; (a) the superelement in the X–Y coordinate; (b) the superelement in the ξ–η coordinate with added parts; (c) the superelement in the ξ–η coordinate with added parts cancelled out; (d) discretized superelement.

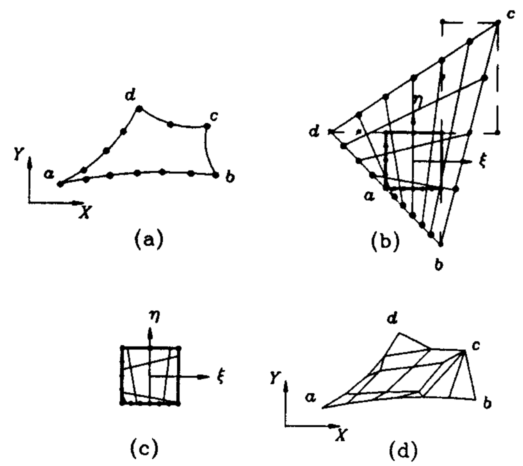


Fig. 5. Procedure of canceling added parts.

2.2. *Two-dimensional superelement with different number of nodes on all sides*

When all sides have different number of nodes, after transformation to the ξ–η coordinate, the two sides which have the smallest number of nodes will be extended from both sides, such that a quadrilateral with equal number of nodes comes into existence (Fig. 4c). As can be seen, when the differences between the number of nodes on all sides are more than specific values (it depends on the number of nodes on both sides), the angle *dab* becomes greater than 180° and the transformation is not possible. In this case, depending on the number of nodes on the two opposite sides and the difference between the nodes on these two sides, as soon the angle *dab* becomes equal to 180°, the side with smaller number of nodes is extended from the other direction to prevent the increase of this angle (Fig. 4d). After transformation to the ξ'–η' coordinate and discretization, it is transferred back to the ξ–η coordinate and the added part is canceled out (Fig. 5). The final discretization in the Cartesian coordinate is obtained by another transformation. As it can be seen in Fig. 5, some of the elements along the sides have a big aspect ratio, which require specific attention and refinement.

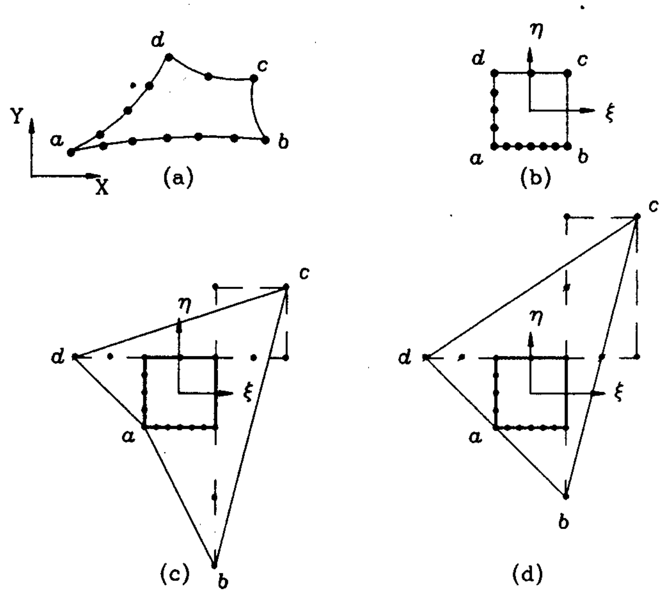


Fig. 4. Procedure of cancelling added parts; (a) the superelement in the X–Y coordinate; (b) the superelement in the ξ–η coordinate; (c) when the internal angle is more than 180°; (d) expanding the side from other direction.

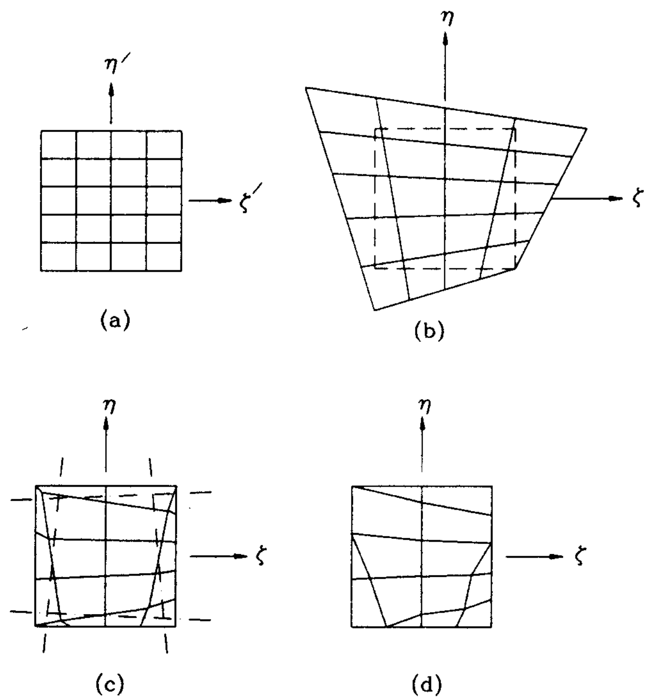


Fig. 6. Refinement procedure of the element.

3. REFINING THE GENERATED MESH

As was mentioned, while in the ξ–η coordinate the added parts are canceled out, one may get some element with a big aspect ratio. If we get such elements, all the nodes which are too close to the sides, which usually cause elements with a very big aspect ratio, should be canceled out. In fact, they should be transferred to the primary nodes on the sides.

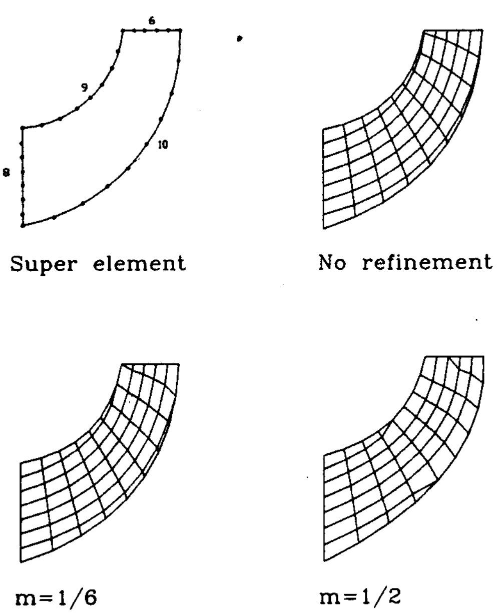


Fig. 7. Refining the generated mesh with different tangent coefficients.

The closeness can be defined as the distance between the sides and a specific line. This line can be defined in different ways. For example, this line can be defined as a line which passes through 1/3 of the length between the nodes on the corner and its adjacent nodes on two connected sides (which we call relative tangent). Therefore, all the nodes between this line and the corresponding side would be transferred to the primary nodes. Figure 6 illustrates this procedure, whereas Fig. 7 shows the refinement for a general superelement when the relevant tangent coefficients are equal to 1/6 and 1/2, compared to the nonrefined superelement.

4. THREE-DIMENSIONAL MESH GENERATION

In three-dimensional cases, by curvilinear isoparametric mapping, the superelement is transferred into a cube in the ξ–η–ζ coordinate (Fig. 1). When the number of nodes are equal on opposite sides, discretization is carried out by ξ, η and ζ constants. Then it is transferred to the X–Y–Z coordinate by eqn (1). When the number of nodes are different on the sides, the above procedure is not applicable. For a different number of nodes two general cases would occur. First, when all the number of nodes on the sides of one plane, i.e. base plane, are different, while the number of nodes on the four sides are equal, knowing that the opposite plane to the base plane has the same number of nodes as the base plane. The other case is when all the nodes of the four sides are also

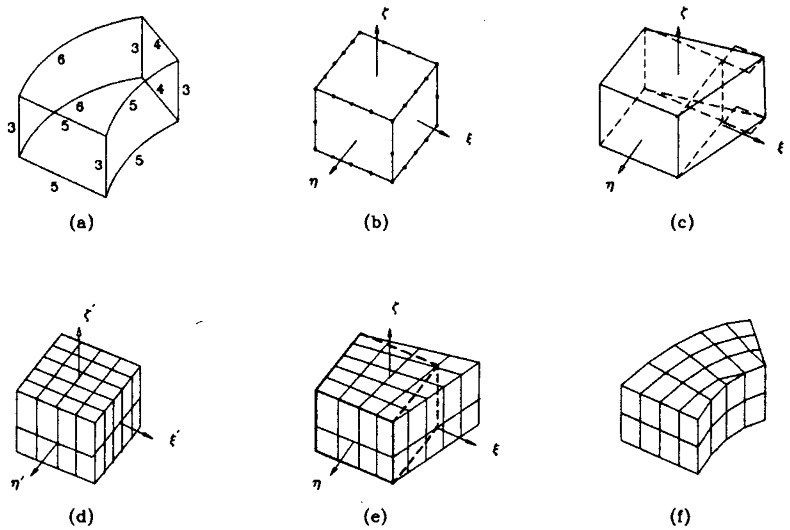


Fig. 8. Procedure of discretization when the number of nodes on the base plane are different.

different from each other. For compatibility between the superelements, only two opposite planes of the superelement should have equal number of nodes. It means that, while the number of nodes on opposite sides of the base plane are different, the plane which is opposite to the base plane has the same number of nodes as the base plane.

4.1. *The number of nodes on the sides of one plane are different*

Suppose that the number of nodes on all the sides of one plane are different (we call this plane the base plane), and the opposite to the base plane has the same number of nodes as the base plane (Fig. 8a),

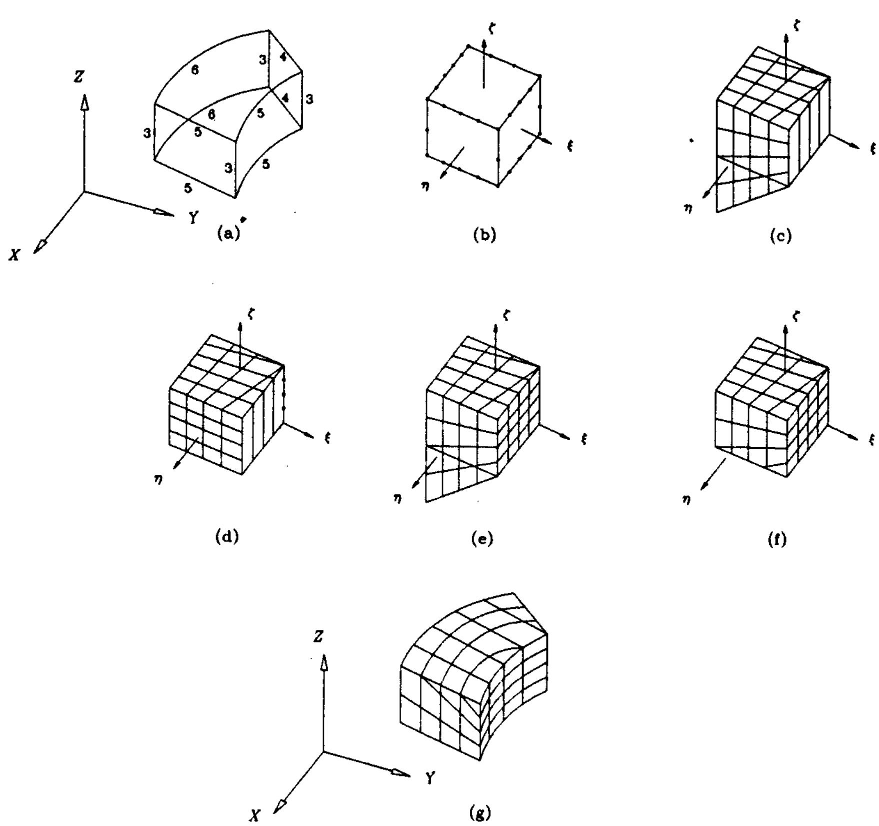
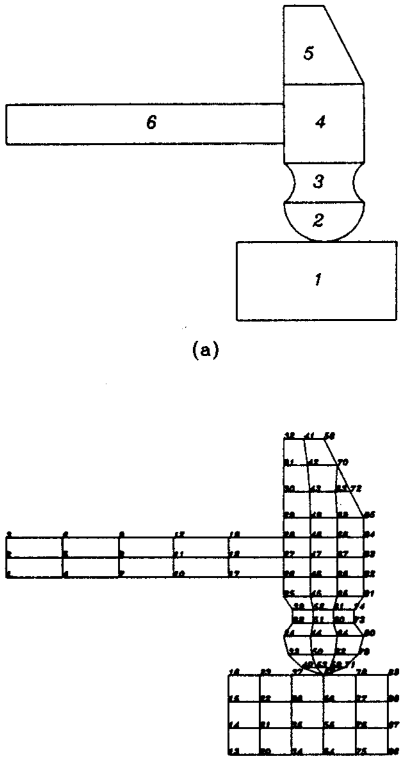


Fig. 9. Procedure of discretization when the number of nodes on all sides are different.



(a)

(b)

Fig. 10. Discretization of objects contacted at a point.

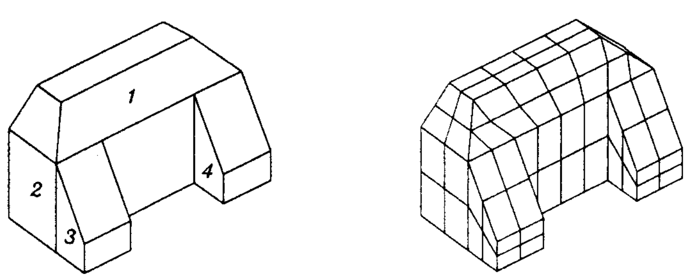
because all the sides which are parallel to the coordinates have the same number of nodes, by applying a method similar to the one discussed in two-dimensional case, the base plane and the plane which is opposite to it is discretized. It means that, first, they are extended in the ξ-η-ζ coordinate, then they would be transferred to a cube in the ξ'-η'-ζ' coordinate (Fig. 8d). Because all the sides have the same number of nodes, discretization is applied. This procedure is shown in (Fig. 8a–f). The cube is transferred back to the ξ-η-ζ coordinate and after refinement, the final transformation to X-Y-Z coordinate is carried out.

4.2. *Base plane and all the other sides have different number of nodes*

When all sides have a different number of nodes (Fig. 9), as before, by ignoring the number of nodes on all the four sides (suppose all the four sides have only two nodes), the base plane is discretized. After discretization of the base plane, the number of nodes on the sides should be considered. Then in the ξ-η-ζ coordinate, the side with the smaller number of nodes is extended such that the number of nodes on all sides become equal. The new hexahedron in the ξ-η-ζ coordinate is transferred to the ξ'-η'-ζ' coordinate. In this coordinate, all the sides which are parallel to the η' axis have the same number of nodes, and by η' = const., the discretization would be completed. After discretization, the cube is transferred to a prism in ξ-η-ζ coordinate. In this coordinate, the added part would be canceled out and refinement is completed. Refinement is the same as in two-dimensional case with the exception that the tangent line is changed to a plane.

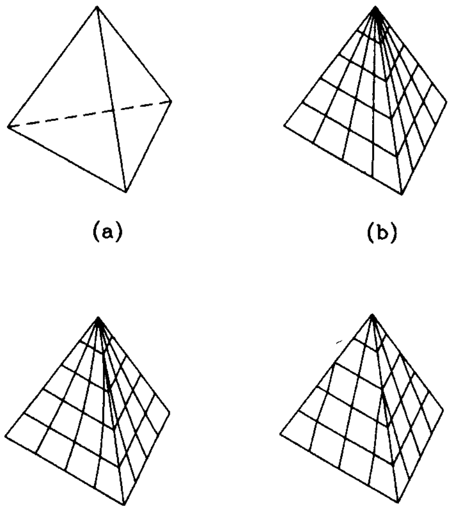
5. COMPUTER PROGRAM

Based on the above method, a computer program is developed. A mesh of superelement is defined by the user in the same way as in ordinary isoparametric methods, with one exception that the user is not to be worried about the number of nodes on the opposite sides of the superelements. A typical superelement mesh is shown in Figs 10 and 11 for two- and three-dimensional cases. As can be seen, superelements 2 and 5 in Fig. 10 have a different number of nodes on their opposite sides. Superelements 1 and 2 are contacted in one point and superelements 4 and 6 are contacted at part of their sides. Superelement 1 in Fig. 11 has a different number of nodes on opposite sides of a plane, and superelements 3 and 4 are in contact with superelements 1 and 2 at part of plate. *In Fig. 12 the number of nodes on all sides of the tetrahedron are equal while the number of elements are different. This is a unique property of this program which by defining a hypothesis side or plane enable one to have a different number of nodes while keeping the number of nodes on the sides equal.*



(a) (b)

Fig. 11. Discretization of a general three-dimensional object; (a) coarse mesh; (b) fine mesh.



(a) (b) (c) (d)

Fig. 12. A general triangular tetrahedron superelement.

Based on the work of Collins [6], the program is furnished with an automatic bandwidth reduction which can work for contacted objects too [7] (Fig. 10).

6. CONCLUSION

With a simple and effective method, a new two- and three-dimensional mesh generation algorithm is proposed. This method is general and applicable for any two- and three-dimensional objects. The method is applicable even for objects which are contacted at just one point. This new approach looks promising, and can be applicable in different algorithms.

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