A New Versatile Two Dimensional Mesh Generation

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Summary

A new simple and effective method for two dimensional mesh generation is proposed. This method is based on isoparametric concept. Superelement can have different number of nodes on all of their sides, this is done by an extra transformation, therefore it is effective for complex shapes, especially for the superelements which has different sizes on their sides and user wants to have almost a uniform elements along the super element. This method looks promising and it can be used in three dimensional cases too.

1.Introduction

Recently a great deal of efforts have been devoted to the development of two and three dimensional generators[1]. One of the methodes is interpolation mesh generation[2],[3].

2.Concepts and definitations

The generation of finite element meshes by mapping is based on isoparametric mapping concept[3]. If the nodal coordinates of the super element (x_i, y_i), are known, the cartesian coordinates of any specific point can be calculated from;

$$x=\sum_{i=1}^{8} N_i(\xi,\eta)x_i \qquad (1)$$

$$y=\sum_{i=1}^{8} N_i(\xi,\eta)y_i \qquad (2)$$

where N_i is the so called shape function[4].

Because the isoparametric element is discreticized by lines of ξ and η=constant, the number of nodes in the opposite sides should be equal. This fact makes some restriction on the mesh generation such as;the number of nodes can not be different on the opposite sides. Therefor when the opposite sides of the supper element have differenr sizes, the element would have smaller sizes in the smaller side or we have to change the supper element into smaller super elements, which will take longer time and make it boring and inconvenience for the user. It should be mentioned that in some cases it is even impossible to discreticized it by using more superelements, unless the number of nodes be equal on opposite sides. This usually happen when we are going from an area with larger elements to the area with smaller elements. Therefor if the number of nodes on the opposite sides of the supperelement can be different from each other, the above difficulties can be easily avoided and it would be more effective in the medium with complex shape and much simpler for the user too.

2. Super element with different nodes on the sides

When the super element has different number of nodes on the sides, two cases may occur. One is when the number of nodes on only two different sides are different and the othere is when the number of nodes on all the sides are different.

2.1 Supper element with different number of nodes on two opposite sides

When the number of nodes on only two opposite sides of the isoparametric superelement is different, as in the case that when the number of nodes on the opposite sides are equal, all the sides would be divided into their required nodes in the $\xi - \eta$ coordinates(which we call them original nodes). The size of the side which has smaller number of nodes would increase from both side, such that the number of nodes becomes equal to the number of nodes of the opposite side. Although the square in $\xi - \eta$ coordinate is changed to a quadrilateral, but the opposite sides have equal number of nodes(fig.1.a-b).It should be mentioned that the side of smaller number of nodes can be extended from one direction too, but it would make the final mesh to incline into that direction. This new quadrilateral which has equal number of nodes on opposite sides can be discreticised by conventional mapping in $\xi_{\prime}-\eta_{\prime}$ coordinate. After discretization, it would be transfered back to $\xi-\eta$ coordinate by

$$\xi=\Sigma_{i=1}^{8}N_i(\xi',\eta')\xi_i \qquad (3)$$

$$\eta=\Sigma_{i=1}^{8}N_i(\xi',\eta')\eta_i \qquad (4)$$

Because of the added part, this discreticised superelement is not the real super element unless, the added part be canceled out. For this purpose the nodes

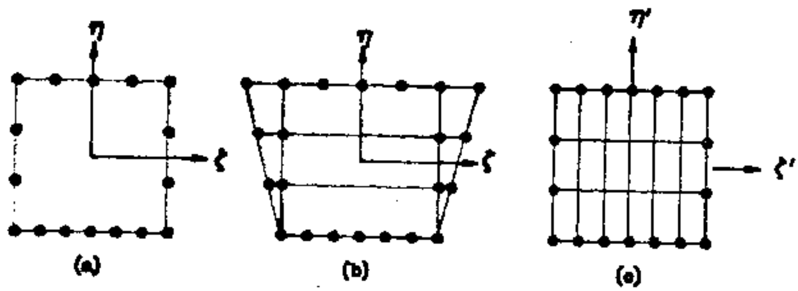

Fig.1 (a) Typical super element when two opposite sides have different number of nodes; (b) when the side with smaller number of nodes is extended; (c) the super element in the $\xi'-\eta'$ coordinate

which are outside of the square in $\xi-\eta$ coordinate should be moved to the original nodes on the sides of the square(fig.2.a), or in the other word, the $\xi-\eta$

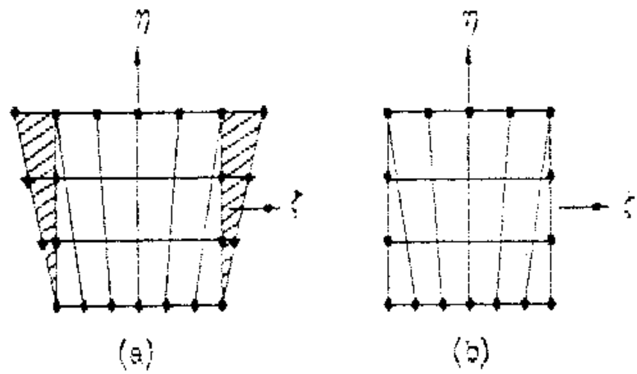

Fig.2 (a)The super element in $\xi-\eta$ coordinate with the added part; (b) the super element in $\xi-\eta$ when the added part is canceled out coordinates of the nodes which are outside of the square would be changed to +1 or -1.

2.2 Supperelement with different number of nodes on all sides

When all sides have different number of nodes, the procedure is the same as in pare 2.1 with some exception in the extention of the sides and in the cancellation of the added part. In some cases, when the difference between the number of nodes are more than some specific numberes, after the expansion of the sides, the internal angle of the quadrilateral would become bigger than 180° and the transformation of this quadrilateral to a square is not possible. In this case by expansion of the side on the other direction, this difficulty would be encountered(fig.3).

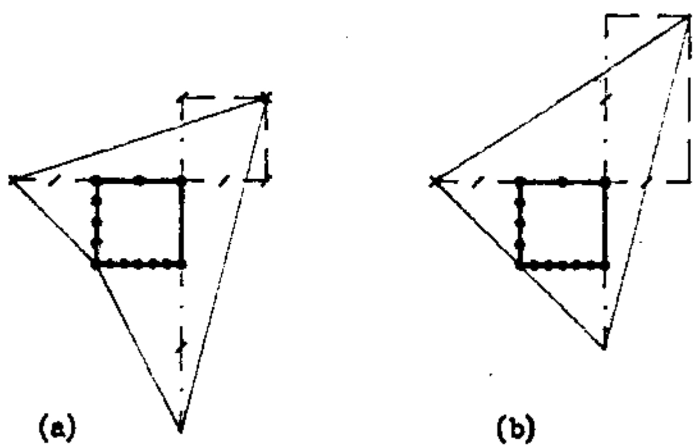

Fig.3 (a) When the internal angle becomes more than 180° (b) expanding the side from other direction

3.Refining the generated mesh

When the added part is cancelling out, sometimes we have a very small elements with respect to the other element on the sides of the super element, therfore the elements which have common side with the superelement, would be checked out, and if their sizes are smaller than a specific number; such as α; the element would be ignored, considering this fact that the number of nodes on the sides should be kept constant. This procedure is shown in fig.4

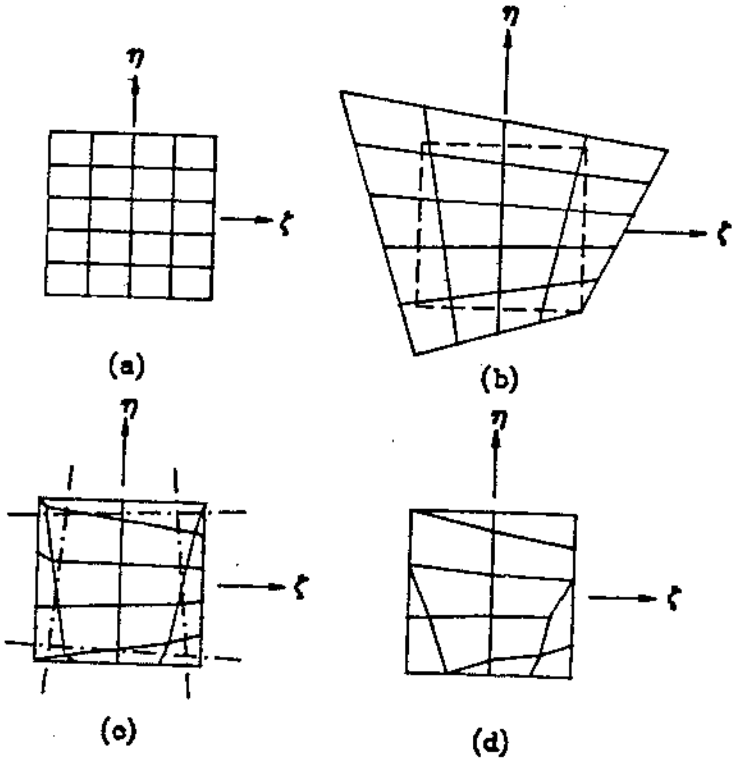

Fig.4 The procedure of refining the elements on the sides from (a)-(d)

3.Computer program

Based on the above procedure, a computer program is written. A mesh of superelement is defined by the user in the same way as in a normal finite element generator, with one exception that the user is not to be worried about the number of nodes on the opposite sides. A typical superelement mesh is shown in fig.5 (straigh-line plotting of the parabolic superelements). As it can be seen superelement 1 and 2 have different number of nodes on their opposite sides. These superelements have both quadrilateral and triangular elements and the triangular element is only on the sides. The reason for this phenomena, as was mentioned before, is in the cancellation of the added part and also may be in the refining procedures. The program is written in such a way that it is

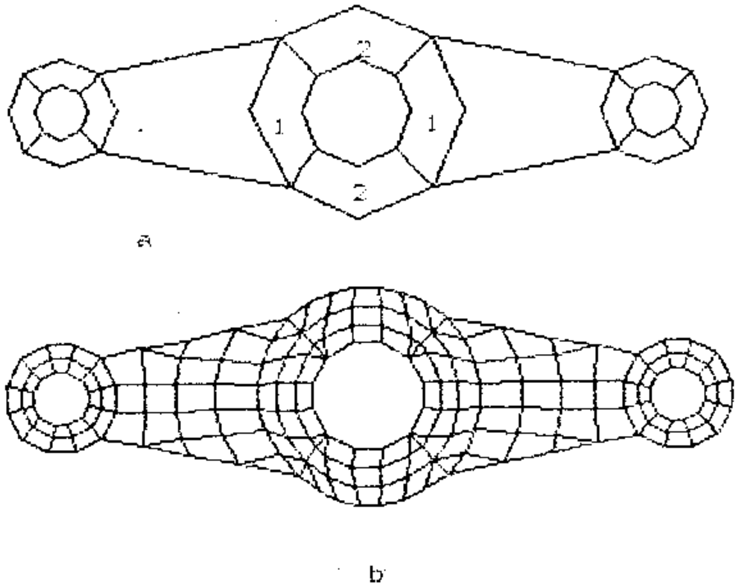

Fig.5 (a) Superelement mesh; (b) General mesh

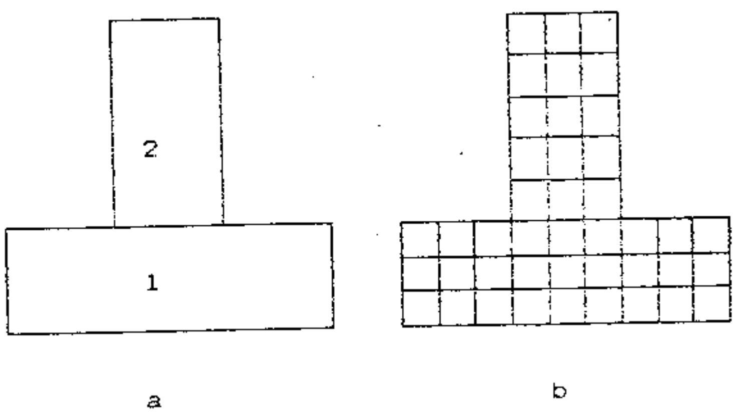

Fig.6 Compatibility between the superelements

flexible in compatibility between the superelement. This fact is shown in fig.6.

The triangular super elements can be discreticised in different ways by giving them different number of nodes to their hypothesis fourth side, which is in fact one of it's apex. As is shown in fig.7, apex

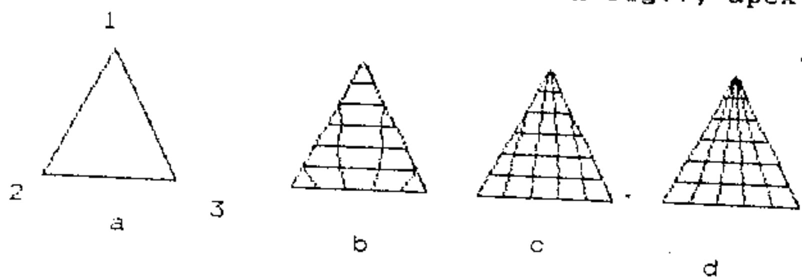

Fig.7 (a)Triangular super element, appex 1 acts as the hypothesis fourth side; (b)-(d) General mesh while the number of nodes are changing in side 2-3

one, which in this case is supposed to act as a hypothesis fourth side, is assigned different number of nodes, while the number of nodes on the other sides are kept constants. It is obvious that if the other apexes are considered as the hypothesis fourth side, the shape would be completely different.

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