D.A.R.K.

Mechanical design of rocket motors

by

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PREFACE

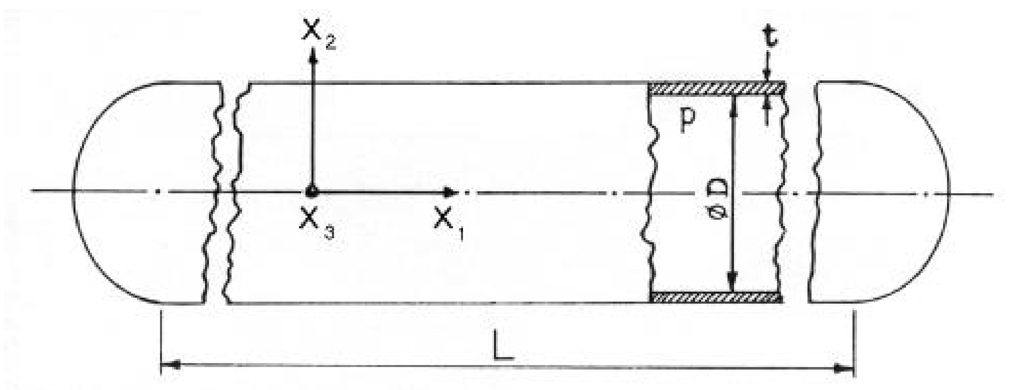
The primary function of the rocket motor usually is to supply a given propulsive impulse, and the designer strives to achieve the impulse requirements with a rocket motor of minimum weight. For this purpose we have applied a simple construction method in our club,with great succes.

Furthermore we have used the method on rocket motors which have failed and it predicted the correct mechanical failures.

I. Chamber Dimension.

The mechanical components of a rocket motor are mainly comprised of the combustion chamber and exhaust nozzle. The combustion chamber is essentially a pressure vessel containing the propellant, which must be able to withstand the high pressure and temperature of the combustion products.

Consider the closed cylindrical vessel subjected to an internal pressure p, as shown in Fig(IA).



Fig(IA).

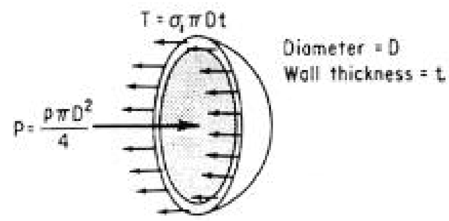
A free-body diagram of a portion of the cylinder cut about a circumferential line, Fig(IB), indicates an external pressure force to act on the closed end and an internal axial tensile force to act within the wall. The tensile force in terms of the average longitudinal stress σ_1 across the severed circumferential area A is as follow on next page.

Thus we have :

$$\sigma_1 A = \sigma_1 \pi D t \qquad (1)$$

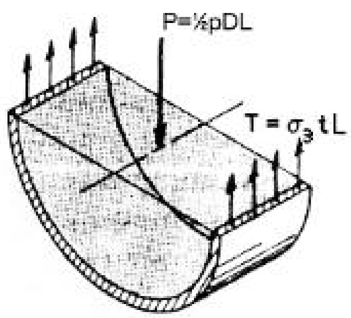
To maintain equilibrium in the longitudinal direction, the tensile force and the pressure force must be equal.

$$\sigma_1 \pi D t = p \frac{1}{4} \pi D^2 \Leftrightarrow$$
$$\sigma_1 = \frac{pD}{4t} \qquad (2)$$



Fig(IB).

To arrive at an expression fore the circumferential stress σ_3 , a second body is sketched as shown in Fig(IC).



Fig(IC).

This is drawn to include the portion of the pressurized gas or liquid contained by this segment of the cylinder. The forces in the vertical direction include a pressure force equal to the product of the internal pressure p and the area LD, and tensile forces on each of the two severed areas.

The tensile force in terms of the circumferential stress σ_3 on the severed area A is:

$$\sigma_3 A = \sigma_3 L t \qquad (3)$$

To maintain equilibrium, then :

$$\sigma_3 Lt = \frac{1}{2}pLD \Leftrightarrow$$
$$\sigma_3 = \frac{pD}{2t} \qquad (4)$$

Since the circumferential stress is twice the longitudinal, the cylinder would fail, burst or explode by splitting up longitudinally. Last we find the average radial stress σ_2 as :

$$\sigma_2 = -\tfrac{1}{2}p \qquad (5)$$

As yield criteria we use the "von Mises" which is given by the equation:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_v^2 \qquad (6)$$

where σ_v is the tensile yield stress of the vessel material.

To insure purely reversible elastic deformation, that is to avoid any lasting plastic deformation, we define a safety factor S_f as :

$$S_f = \frac{\sigma_v}{\sigma_{ref}} \qquad (7)$$

where σ_{ref} is a reference tensile stress in terms of, when $\sigma_{ref} = \sigma_v$ yield conditions is reached, when $\sigma_{ref} < \sigma_v$ we have purely elastic deformation and when $\sigma_{ref} > \sigma_v$ we have purely plastic deformation.Quality tubes for hot steam applications uses a safety factor of app. 1½ as international standard. Also standard is to take account for weakening phenomena as welding on the pressure vessel, which is the case when the rocket motors aft closure is welded to the chambers end.

As precaution we involve a strength-factor z:

$$z = 0.9 \qquad (8)$$

when proper materials are used, that is use of hot gas pressure vessel steels, such as DIN 35-8/III & 45.8/III just to mention some readily availiable common steels.

The connection to the safety factor is then specified as

$$S_{fz} = zS_f = \frac{z\sigma_v}{\sigma_{ref}} \qquad (9)$$

The equation (6) the von Mises-criteria, then takes the form of:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\left(\frac{z\sigma_v}{S_f}\right)^2 \qquad (10)$$

Whereas either the max. pressure for a given vessel or the min. wall thickness t for given pressure and diameter is seeked in the analysis, solving equation (10), by use of (2), (4) and (5), gives:

$$P_{\max} = \frac{4t}{\sqrt{3D^2 + 6Dt + 4t^2}} \cdot \frac{z\sigma_v}{S_f} \qquad (11)$$

$$\begin{aligned} t_{\min} &= \left(\frac{3 + \sqrt{48Q^2 - 3}}{16Q^2 - 4}\right) \cdot D \\ &\approx \left(\frac{1}{4}\sqrt{3}\,\frac{1}{Q} + \frac{3}{16}\frac{1}{Q^2}\right) \cdot D \\ Q &= \frac{z\sigma_v}{pS_f} \end{aligned} \qquad (12)$$

In practice, standard equations for dimension of pressure vessels, are "easy to remember" approximate equations, which usually follows the theoretically based solutions in terms of a fairly constant proportionality-coefficient around 1.25, that is, t_{min} in practice is calculated app. 25% higher than the corresponding theoretically based figure.

Finaly the usefulness of the obtained formula, is depending upon the designers common sense. To insure proper design parameters we must include the tolerance of the manufactured pressure vessels wall-thickness. We have:

$$t_{nom} = t_{\min} + dt \qquad (13)$$

Where t_{nom} is the nominal measure and dt is the guaranteed tolerance of the nominal measure, usually 10% of t_{nom}. This gives us the desired nominal measure to design upon:

$$t_{nom} = t_{\min} + 0.1 t_{nom} \Leftrightarrow \qquad (14)$$
$$t_{nom} = 1.11\ t_{\min}$$

To find the proper tube, one must investigate the available possibilities and choose a nominal wall thickness that lies as close as possible over the calculated nominal found measure (14).

<u>EXAMPLE:</u>

A zinc/sulphur rocket motor is to be dimensioned. The inner chamber diameter is fixed to:

D =ø100.0 mm

The maximum expected pressure is known to be

P = 130 atm

The pressure vessel is in DIN 35.8/III which gives a tensile yield stress of:

σ_v = 35kp/mm^2 * 0.8 ≈ 275 Mpa

The safety factor is S_f =1½ and the strength factor is

z = 0.9

and finally the tolerance follow. (14). Find t_{nom} ?

<u>Ans.:</u>

(12) gives: t_{min} = 0.03528D, and

(14) gives: t_{nom} = 0.03916D = 3.916mm ≈ <u>4.0mm</u>

II.Bolted joints.

Consider the common used method of fixing the exhaust nozzle to the combustion chamber with a bolted connection as shown in Fig(IIA). The problem is now to find the necessary dimension and number of bolts, and finally their placement on the combustion chamber.

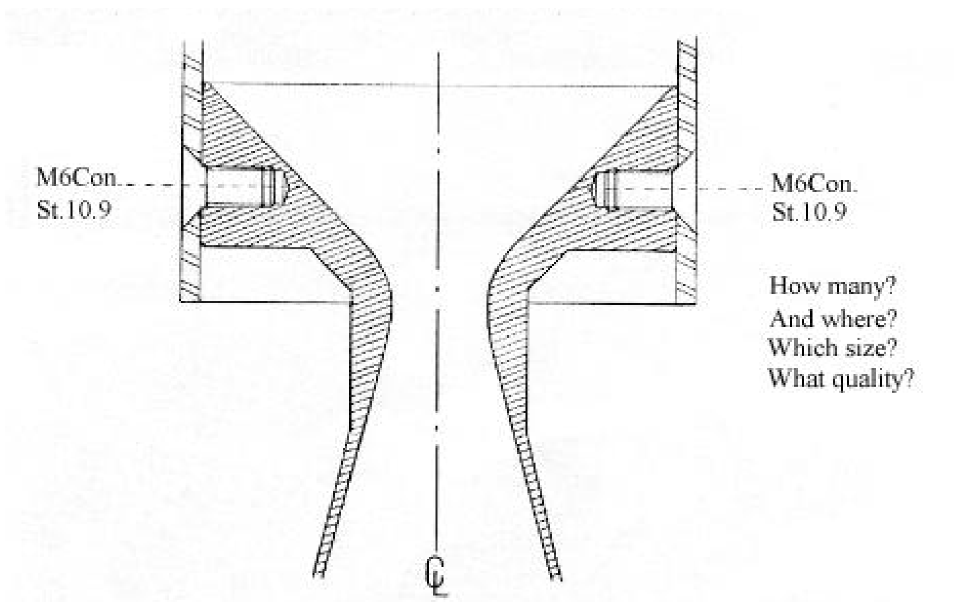
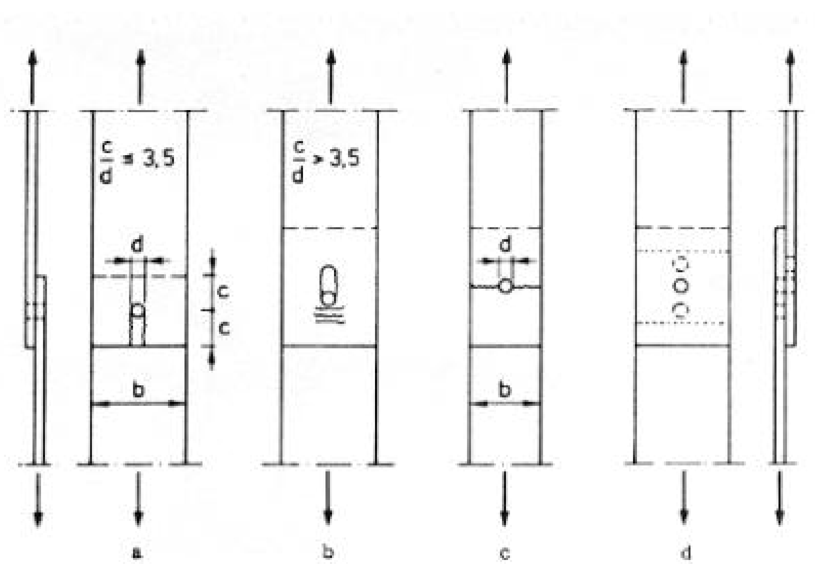


FIG (IIA).

It is customary to consider the strength of a bolted connection to depend upon three quantities the crushing or bearing strength of the connected plates, the tensile strength of the connected plates, and the shearing strength of the involved fixation bolts. It is presumed that the joint will ultimately fail in one of the four ways shown in Fig(IIB).



FIG(IIB).

The formulas is originally derived for plates, but we assume that they also applicable for a cylindrical shaped vessel. Further we neglect any friction between the cylindrical vessel and the exhaust nozzle.

In bearing Fig(IIB-a,b), failure is caused by the pressure force between the cylindrical surfaces. We have:

$$P_f = 2ct\frac{\sigma_v}{\sqrt{3}} \quad \text{for} \quad \tfrac{c}{d} < 3\tfrac{1}{2} \qquad (15)$$

$$P_f = 4\tfrac{1}{2}td\sigma_v \quad \text{for} \quad \tfrac{c}{d} \geq 3\tfrac{1}{2} \qquad (16)$$

In tension Fig(IIB-c), the strength of the joint depends upon the the tensile strength of the plates and the minimum area that must sustain the force. We have:

$$P_f = a\sigma_v(b-d)t_{\min} \qquad (17)$$

where:

$$a = 1 - 0.9 + 3r\tfrac{d}{b} \quad \text{for} \quad \tfrac{d}{b} < 0.3$$
$$a = 1 \quad \text{for} \quad \tfrac{d}{b} \geq 0.3$$

where b is the distance between two bolts, and r = 1/m, and m is the number of bolts in a row circle.

The shearing stress of the used bolt Fig(IIB-d), is:

$$P_f = 0.32\sigma_b d_b^2 \qquad (18)$$

where σ_b the tensile yield strength of the boltmaterial and d_b is the bolt shaft diameter. The notation σ_v follows the already used, that is, σ_v is the vessel-material tensile yield strength. 0therwise, notation refers to Fig(IIBa-d).

The method now consist in trying to balance the two last mentioned forces (17) and (18), so that both are approximately equal to:

$$
\begin{aligned}
P &= \frac{Cross - section\ Area\ of\ Vessel}{\text{Number of bolts in a row}} \cdot P_{\max} \cdot S_f \\
&= \frac{\pi D^2 P_{\max}}{4m} \cdot S_f
\end{aligned}
\qquad (19)
$$

where the safety factor is usually as before mentioned - 1½, and P_{max} is the maximum expected pressure.

The variable parameters are m=number of bolts, d_b = shaft diameter of the bolts, σ_b =tensile yield strength of bolt-material used.

If metric system by convenience is used, following possibilities are avaliable:

m = 3, 4, 6, 8, 10, 12, 14, 16.....
d_b= M4, M5, M6, M8, M10,MI2......
σ_b = (6.8), (8.8), (10.9), (12.9)... No other.

where quality (10.9) means :σ_b = 100kp/mm^2 * 0.9 ≈ 880 Mpa by standard DIN-convension.

When (17) and (18) are satisfactory balanced to (19), the bolt row mounting distance c, can be derivied from either (15) or (16). If bolts with 45^o conical heads are used, because of best stress distribusion, usually (15) will be the case. This gives the expression for c:

$$
c = \frac{P_f \sqrt{3}}{2t\sigma_v} \qquad (20)
$$

A convenient straigth measure over the calculated c figure, is then choosen as the optimal distance to the chamber end.

III. An example-Bolt joints on Mini-DARK.

The rocket motor Mini-DARK is the smallest standardised Rocket motor to reach fligth dynamically in DARK, about 12 are planned - 4 have been made and tested.

The pressure vessel cross section dimensions are:

D_i = ø70.0mm
t_{nom} = 2.5mm ± 0.25mm

The steel is DIN St-35.7, which gives :

$\sigma_v = 35kp/mm^2 * 0.7 \approx 240Mpa$

After a few computations of different combinations of m, size and quality, one will find that the combinations:

m = 6
d_b= M6 Conical head. (Means d becomes bigger)
$\sigma_b \approx 1060Mpa$ (Means quality 12.9)

gives a good balance between (17) and (18) and furthermore gives the highest value of P_f meaning a better safety factor when resembling P_f with P (19).You get following figures:

(17) P_f = 15150 N.
(18) P_f = 12200 N.

this means that P_f (18) is the critical force, to find the safety factor we get :

(19) P = 8350 N. & $S_f = P_f$ (18) = 12200 N. ⇔

$$S_f = \frac{12200}{8350} \approx 1.46$$

which is quite satisfactory, because this the same value the pressure vessel is designed with.

But if one consider the different failure situations, in terms of which of these that appear to be least dangerous, -least damaging to materiel, one will find that a Nozzle Blowout is preferable. No damage is done on the often costfull pressure vessel only the bolts suffers and from a safety point of view we know the blowout direction. This means one can, without great measures - increase the safety!

To execute this concept, clearly the safety factor of the shearing stress of the used bolt P_f (18), must be smaller than the other forces safety factors included the pressure vessels safety factor. This can effectively be accomplished by using a quality (10.9). bolt instead of (12.9), reducing safety factor of P_f (18) with app.17%. In the actual example this gives us:

$S_f = 1.22$

The distance c is now left to calculate, remembering that P_f (18) must have the least safety factor, P_f (15) is bound to equalise with P_f (17):

$P_f (15) = P_f (17) = 15150$ N. $\Leftrightarrow$
$c \approx 24.30$ mm $\approx 24½$ mm

NB!- <u>Note about Safety Factors.</u>

The factor of safety is a term defined as the ratio of the stress necessary to produce failure to the working stress. As mentioned, we normally use a factor of safety of 1½ which means that the load could be increased 1.5 times before failure would occur. The need for this safety margin is apparent for many reasons: stress itself is seldom uniform; materials lack the homogeneous properties theoretically assigned to them; abnormal loads might occur; manufacturing processes often impart dangerous stresses within the component. These and other factors make it necessary to select working stresses substantially below those known to cause failure.

The Aft Closure-Welding Rules.

Two caracteristic ways of sealing the pressure vessel in the opposite end of the nozzle, are either to bolt a metal pressure retainer disc to the vessel similar to the nozzle, or to weld a retainer disc to the vessel. These are the common used principles by amateurs. The possibility of using a bolted joint can often be convenient but is also the most heavy solution. As the mechanical conditions are identical to those of bolted joints of the nozzle, exactly the same formulas should be applied in this case. We therefore concentrate about the solution involving welding the aft closure to the pressure vessel wall.

Certain rules/restrictions should be listed as below:

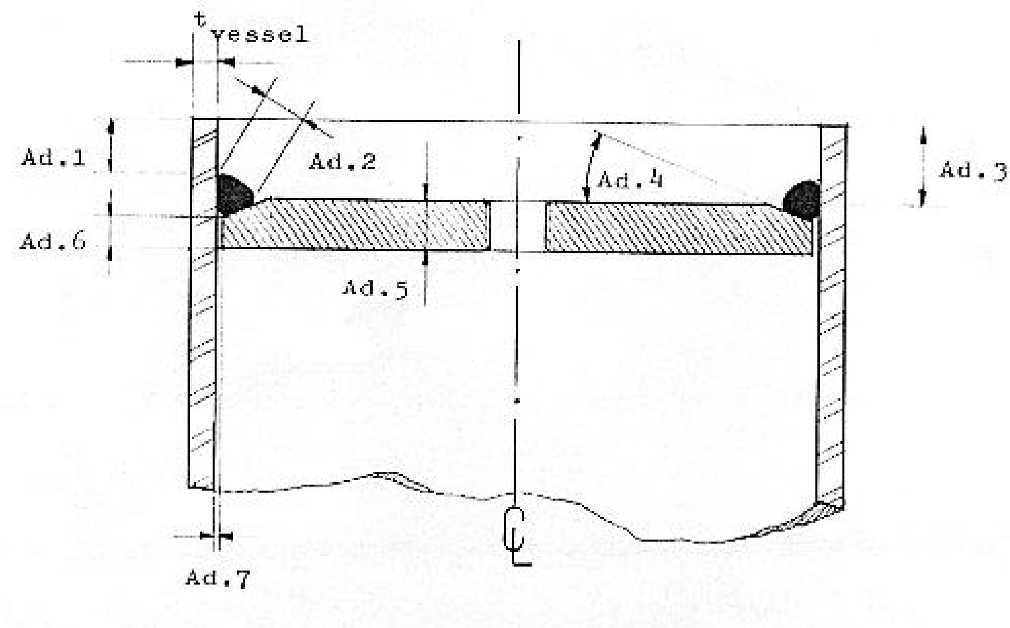
a. Closure material must be identical to vessel material.

b. Welding electrode must correspond to the above mentioned material, by standards specified.

c. Rules listed, are only valid for t_{vessel} > 2mm.

d. If any penetration of the aft closure is requested, penetration shape should be circular or ovals. Furthermore must the surrounding edge region at the penetration be reinforced either by bolting a flange or during aft closure manufacture leave surplus thickness around the closure penetration.

If the above mentioned can be considered fulfilled, no problems should be encountered by using the following contruction/welding instructions. On next page a typical welding situation is shown in Fig(IIIA), and the measures seeked designated by Ad."i", are listed below.



Fig(IIIA).

We begin with :

Ad.1 From upper edge of welding-seam to end of pressure vessel, distance should be : ≥ 2.0 t_{vessel} ,where t_{vessel} is the vessel wall thickness.

Ad.2 Welding-seam should be : 2½-3*t_{vessel} wide.

Ad-3 From the outwards facing surface of the closure, to end of pressure vessel distance should be ≥ 3*t_{vessel}

Ad.4 The welding gap angle,that is angle of the welding cone on the closure, is: $\theta_w = 30^0$

Ad.5 The closure disc thickness should be ≥ 2*t_v

Ad.6 From the welding cone outer edge to inwards facing surface of closure, distance must be:≈ 1.0*t_v

Ad.7 The gap between pressure vessel and closure disc, should be less than 0.05*t_{vessel}

IV.Nozzle Construction.

In the following, we want to show the method we use to specify certain design parameters for our exhaust nozzles. It is obvious that certain restrictions/limitations of natural reasons occur in the shaping of nozzles with just a minimum of optimisation with respect to weight, performance, heat resistance, reuse and maintainance. One wi11 always strive to achieve low weigth in combination with high performance, though this often confront the designer with fundamental problems such as heat conductivety and erosion phenomenas.

In Fig(IVA) we have a simple exhaust nozzle of the kind used for zinc/sulpliur rocket motors. We start from the inside of the rocket motor and try to limit some parameters as the gas-flow reaches the different important places.

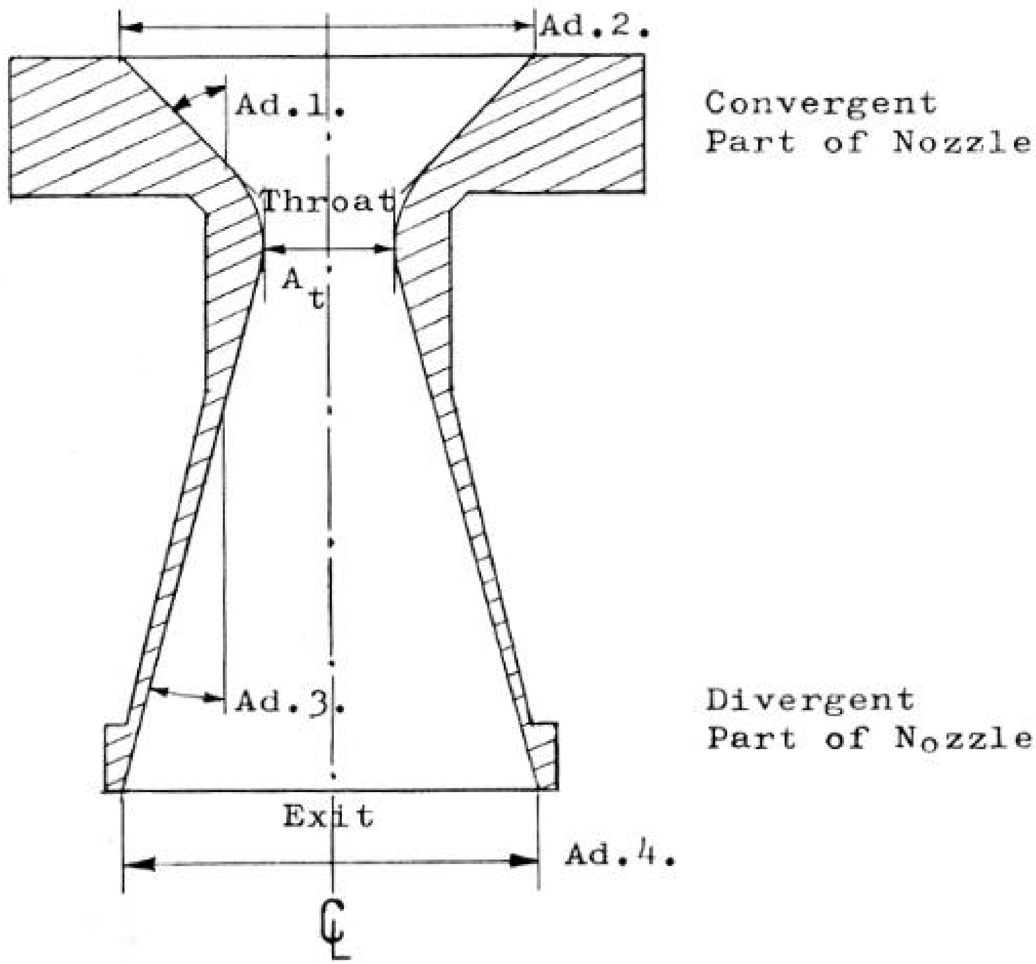


Fig (IVA)

We begin with:

Ad.l. CONVERGENT PART OF NOZZLE-Angle of approach.
Usually lies within: $30^o \leq \theta_{conv} \leq 45^o$,mainly because :

$\theta_{conv} > 45^o$: Gives turbulence,poor stability of gas acceleration lapse,risk of reflection of detonation waves in the chamber.

$\theta_{conv} < 30^o$ Gives much surplus weight and heat looses.

Ad.2. RATIO OF INITIAL AND THROAT AREA-Convergent cone.
Usually lies within : $4 < A_0/A_t < 9$, and proves to insure proper uniform acceleration lapse until gas velocity is Mach 1 at A_t. But if:

$A_0/A_t < 4$ gives improper thermodynamic conditions.

$A_0/A_t > 9$ is waste of usefull space and increases dead-weight.

Ad.3. DIVERGENT PART OF NOZZLE-Angle of exit.
Usually lies within $10^o \leq \theta_{div} \leq 15^o$,mainly because:

$\theta_{div} > 15^o$ energy loses due to increase of normal composant of thrust with increase of θ_{div}

$\theta_{div} < 10^o$ energy loses due to increase of heat conductance to exit cone when θ_{div} decreases.Increasing surplus weigth of exit cone.

Ad.4. RATIO OF EXIT AND THROAT AREA-Divergent cone.

Usually a thermodynamical problem, depending upon the gas composition,its temperature, the chamber pressure and the outside pressure, when proper defined propellants are used.

For zinc/sulphur imperical measures are needed. From experiments/static tests we found:

$8 < A_e/A_t < 12$, is satisfactory for pure Zn/S -3:1 mixture.

These general limits of some important design parameters, all refer to the requirement for high performance, and not much to minimisation of the overall weigth of the nozzlechamber configuration. Therefore we now will show some weigth saving measure-restrictions, mainly dealing with the bolted joint between nozzle and chamber. Beneath the joint section of a typical nozzle chamber arrangement is shown in Fig(IVB).

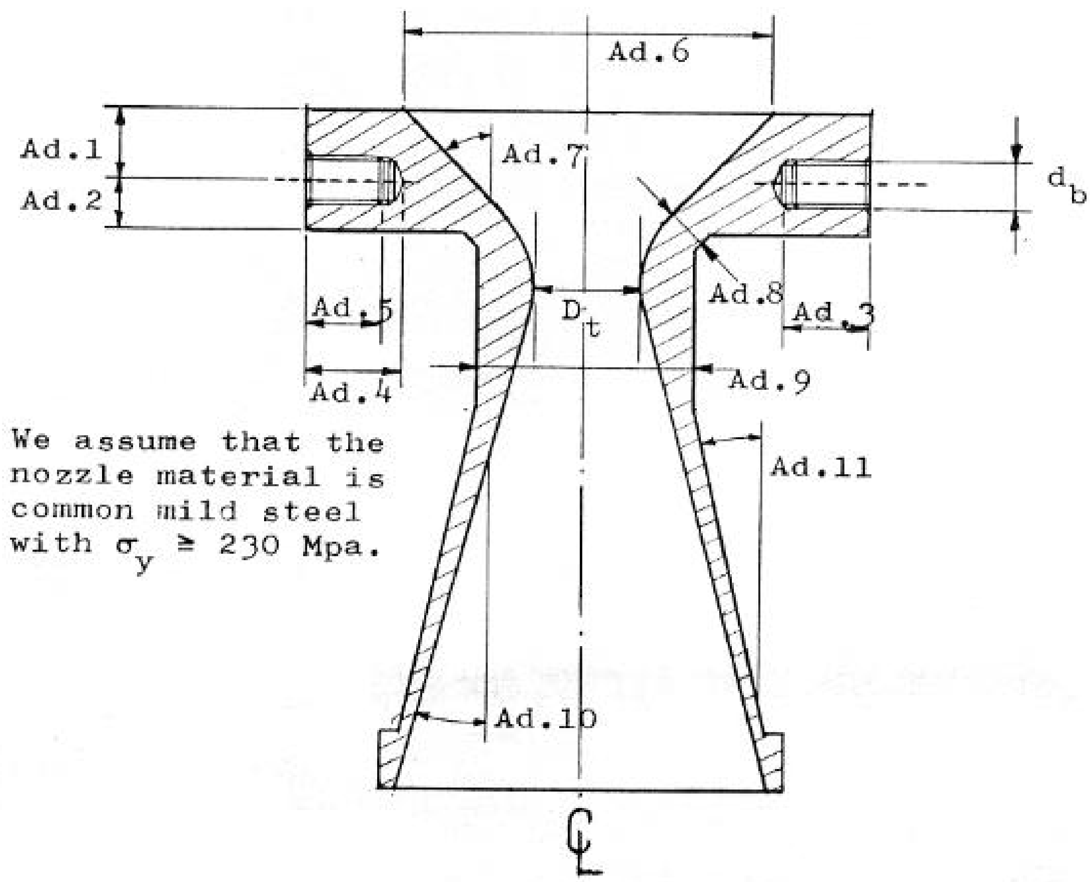


Fig (IVB)

We begin with:

Ad.l. The distance from the center of the bolt joint to the inward edge og the nozzle should be:$\geq 1.5d_b$, where d_b is the shaft diameter of the bolts used.

Ad.2 The distance from the center of the bolt joint to the outward edge of the nozzles cylindrical section should be: $\geq 1.0d_b$.

Ad.3 The min. thread depth should be: $\geq 1.75d_b$.

Ad.4 The min. thread drilling depth should be: $\geq 2.0d_b$, and of course not penetrate the convergent cone wall.

Ad.5 The min. thread length of the steel bolt should be: :$\geq 1.5d_b$.

Ad.6 The initial approach diameter should lie between $2D_t$ and $3D_t$.

Ad.7 The angle of approach should max. allowable: $\theta_{conv} = 45^o$.

Ad.8 The convergent cone wall thickness should be $\geq 0.3\ D_t$, or $\geq 2.0\ t_{vessel}$, (chamber wall).

Ad.9 The cylindrical section at the throat should be: $\geq 1.8\ D_t$.

Ad.10 The divergent angle of exit should be max.: $\theta_{div} = 15^o$.

Ad.11 The outer divergent angle of exit should be: $\theta_{divOut} = 13^o$-14^o.

V. Nozzle throat inserts.

The already mentioned hint of design and simple construction, usually satisfy the expectations of the designer. This is partly due to carefull considerations with respect to several parameters. Therefore the construction criteria so far, are only valid under the following conditions:

a. For zinc/sulphur motors, burning time should'nt exceed 3 sec.

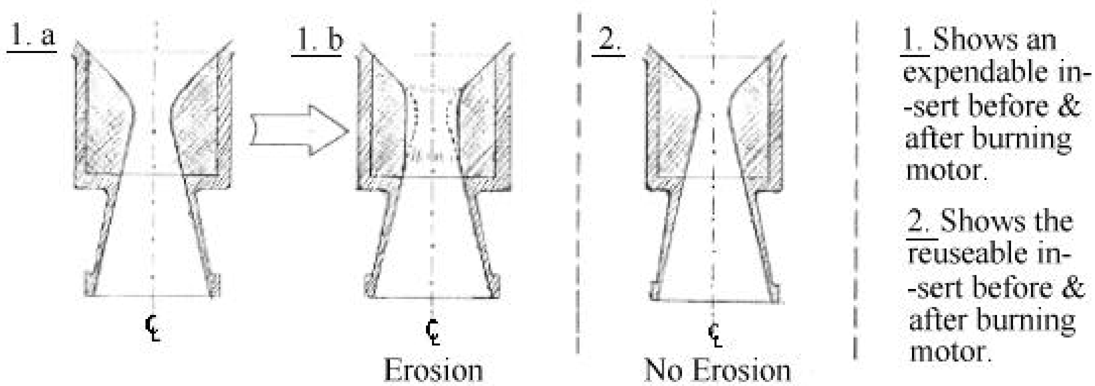
b. Size of joining bolts should exceed app. metric 5mm normal thread and min. DIN 8.8 strength class.

c. Vessel diameter at least 050mm as the the smallest motor tested so far, designed by use of the mentioned theory.

The limit of burning time is necessary because of the increasing risk for erosion phenomena in the throat region, and ultimately a severe drop in the vessel materials tensile yield strength as temperature rises from heat conduction through the chamber wall. this means that high pressure load and high temperature weakening might uccur simultaneously, and this would be conditions beyond the validity of the developed design/construction theory. The case of erosion might lead to partly or even total damage of the exhaust nozzle - confronting the requirement for reuse.

The limits of bolt size and quality are common sense, and the last mentioned are merely the lower region of experience.

To encounter the problem of erosion in the region of the nozzle-throat, an expendable nozzle-throat insert can be designed or a reuseable heat-resistant nozzle-throat can be manufactured. Such precautions would raise the allowable burning time to app.5sec. which could prove to be convenient in some cases. The two different solutions to the problem of course



separates the kind of material used for either of the solutions in two categories.

Fig (VA)

First we take the materials suitable for expendable nozzle-throat inserts:

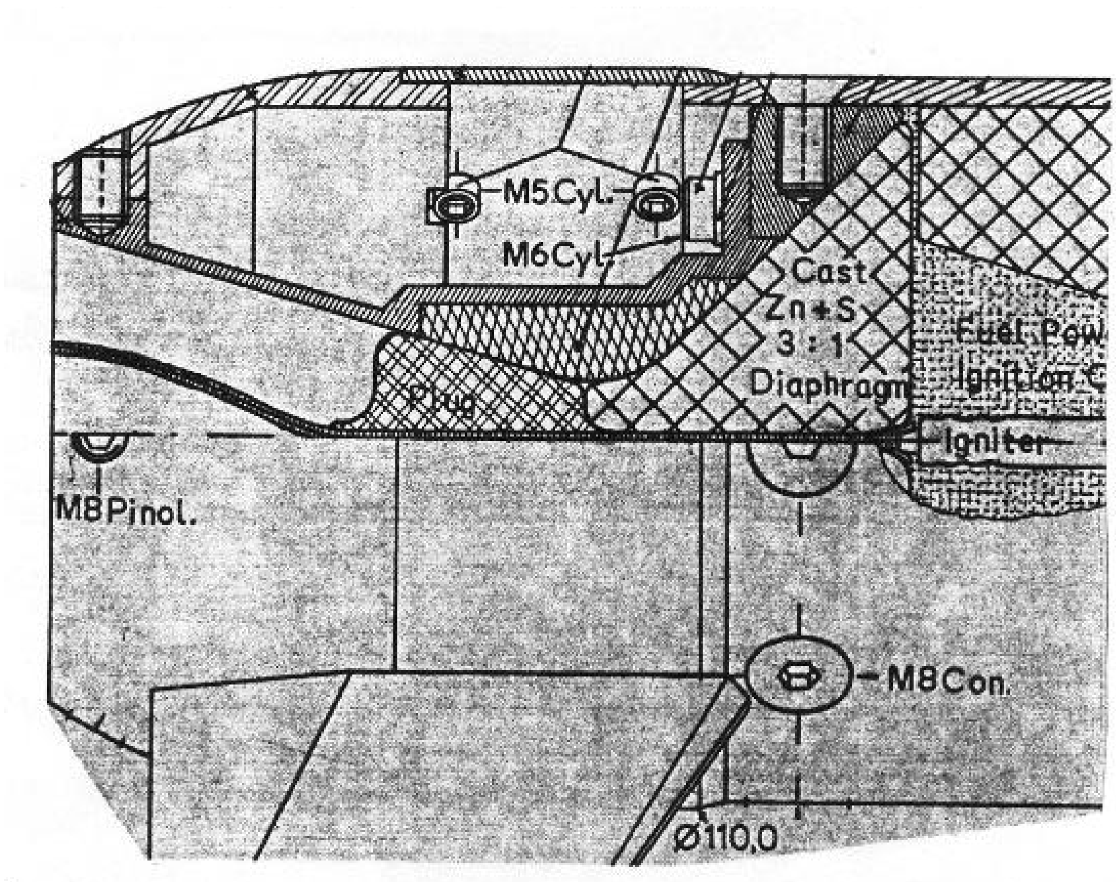
- Ordinary mild steel
- Brass
- Cast iron
- Ceramic

And then the materials suitable for reuseable nozzle-throat inserts:

- Graphite
- Molybdenum
- Tungsten
- Tantalum

The last three mentioned are expensive and difficult to handle due to their high melting point and hardness. Fore us as amateurs these aren't considered realistic to use. We have tried to manufacture some inserts of mild steel, ceramics and of graphite with succes.The case of ceramics we did some further experintents - please see next passage(vi). Another advantage of nozzle-throat Inserts, is that if they are reuseable one quickly can change the exit-throat area ratio, without changing the entire nozzle-construction. This is a usefull tool, when a motor sometimes is used with one kind of propellant and then another kind. This was the case with one of our motors.

Actually it was this motor that caused the need for new ideas fore nozzle-design. The first nozzle fired at app.60% of the designed burning time showed erosion phenomena. So of course a fullscale test was out of question. A new nozzle was made using either an expendable graphite insert or one made of ceramic material, but as these experiments took more valuable time than expected we made a couple of mild steel inserts on a lathe. Due to the increased amount of material in the throat region no erosion appeared at next static test.Below an assembly-drawing of the nozzle-vessel/boattail configuration is shown in Fig(VB).



Fig(VB). Assembly drawing of STYX aft section.

The double Hatched part in center of the drawing is the insert of mild steel and the surrounding part is the insert keaper - the rest of the nozzle unit which is flanged to a retainer ring on the chamber.

VI. Ceramic Nozzle-Throat Inserts.

Why not graphite? As mentioned before, inserts or graphite and ceramic material was made. The graphite version, was manufactured of a solid bar of compressed densified graphite dust, the kind used as electric resistance heating element in high temperature furnace. The cylindrical bar, was turned to the desired shape on a lathe. The advantage of Graphite is its low density of app.1.85 g/cm^3 compared with steel 7.85 g/cm^3. The disadvantage is its low tensile strength, so we decided to use a ceramic nozzle throat insert instead.

Use of Ceramics.

Ceramic material has many advantages such as low permeability towards gas penetration, good tensile strength, at high temperatures and a low density of app. 1.9-2.2 g/cm^3. One of the strongest cerarmic materials for very high temperature purposes, is based upon crystaline Aluminium Oxide, known as the mineral - Corund, and a similar binder. To be specific the basic material used in our experiments with nozzle throat inserts, has been the combination of coarse corund (Al_2O_3) as strengthening agent and fine Calciumaluminate ($Ca_3(AlO_3)_2$) as hydrolytic binder.

The manufacture of the Components.

The fine CalciumAluminate generally symbolised by C_3A in ceramic nomenclature, is made commercially by melting the stochiometrically amounts of CalciumOxide and AluminiumOxide in an electric resistance arc furnace, at high temperature followed by casting the mixture, crushing it and finally sifting it, thus yielding a fine powder with an average grain-size of about 20 μm. This substance has the ability of establishing a strong binding by means of hydrolytic inter-

-action with water, much the same reaction as ordinary concrete undergoes when it hardenes and dryes.

Nomenclature in Ceramics-science.

Standard in ceramics is,to divide the different elements within a ceramic material in a combination of their oxides, conseqently giving the formula: $3CaO \cdot Al_2O_3$ in our case with CalciumAluminate.The point then, is that the above mentioned content of Al_2O_3 in the listed formula, usually is added to the other sources of Al_2O_3 in the ceramic material, such as the pure Al_2O_3 the Corund we are using as strengthening filament represents.

An alumina ceramic material has a limit of 70% w/w Al_2O_3 content as a minimum, otherwise it has no practically importance because of weakening effects due to the increased CaO-content.

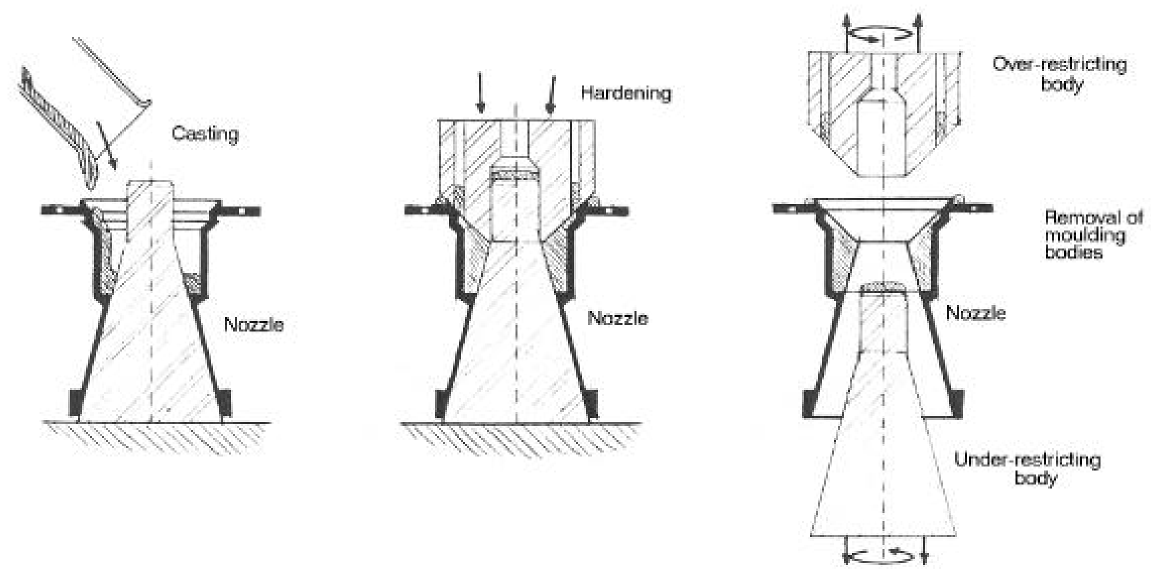
When C_3A then can be divided into ap.62.2% w/w CaO and ap. 37-8% w/w Al_2O_3 it becomes easy to calculate, that a mixture of C_3A and pure A (=Al_2O_3) having a total of min. 70% w/w A, can be obtained by say ap.45% w/w C_3A and 55% w/w A.

3 Primary selection of Ceramic Mixture.

Such a mixture has proved to be a compromise of the request for a strong binding, giving max. allowed C_3A content, and the main objective - the high temperature strength of alumina ceramic material. Lower C_3A percentage will give a non-professional problems such as poor binding, high porosity and extreme low strength, not to mention severe handling problems as well. This was the first phase of the experiments, to find the most convenient mixture of C_3A and A in our case. The next natural step, is the technique of wet casting the nozzle throat insert and the following different heat treatments needed to accomplish a useable item for protection of the rocket motor hardware against damaging high temperature contact.

The wet casting Proces.

At first,one has to decide wether the casting is to take place directly into the cavity within the nozzle, or outside in a mould. Here we fund it basically easier to cast into the nozzle cavity with an under - and an over restrictorbody, enabling us to do the entire insert almost without further preparation. In Fig(VIA) the concept in shown.



Fig(VIA).

The before mentioned percentages of C_3A and A are prepared in the needed amounts for the given nozzle insert.When well mixed dry, small portions of hot water is added,until the consistency just shows fluidability, or with other words as little amount water to insure casting ability as possible. With the nozzle placed fixed to the lower restrictor part, the wet casting can proceed by pooring the pasta like, water mixed ceramic material into the shown cavity. When enough, the cavity filled up, the upper restrictor part is placed by pressing surplus material away and kept in a fixed position while the hydrolytic interaction takes place. This takes min. about 14 hours, and then the restrictor parts can be removed by carefully twisting these around the axis of the nozzle.

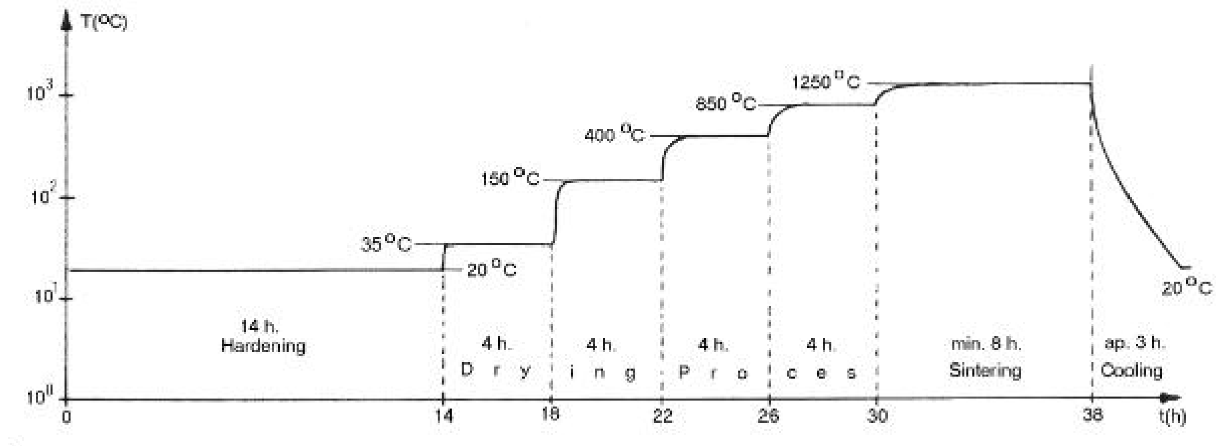
The Temperature Treatments.

Surplus water is then slowly evaporated at ap. 35^{o}C for 4 hours, followed by 2 hours at ap. 150^{o}C to remove the remaining physically bound water in the material.Under this drying proces the casted material will contract sligthly, enough to remove it from the nozzle for further heat treatment at higher temperatures. The binding effect under the hydrolytic interaction with water, is a result of the formation of both CalciumAluminate-hydrates and Aluminiumhydroxide. But these types of binding mechanisms are only valid in the low temperature region and has the advantage of enableing us to do the shaping of the insert at ordinary temperate.

Aluminiumhydroxide decomposes into AlurniniumOxide and water at ap. 300^{o}C so the first treatment consists in ap.4 hours at 400oC. CalciumAluminate-Hexahydrate decomposes into C_3A and water at ap. 700-800^{o}C, forcing us to have a ap. 4 hours treatment at 850^{o}C.

All decomposition processes are now completed, and the next step is the final important sintering of the ceramic material, and at least 1000ºC is needed to allow this. If one has the capacity, 1600ºC must not be exceeded and therefore marks the max. sintering-temp.

Our max. temp. was restricted to ap.1250ºC due to the oven we used. The time needed to complete satisfactory sintering was considered to be at least 8-12hours at this temperature. Afterwards the insert should cool to ordinary temperature in the, same mode as the oven cools, to prevent tensions within the insertmaterial. Below a typical Time-Temperature Diagram is shown in Fig(VIB).



FIG(VIB)

The use of the Ceramic Insert.

The nozzle-insert is now ready fore use, by glueig it with Araldite into the cleaned cavity of the steel-nozzle. If not used instantly, the insert or the entire nozzle-assembly should be kept protected against moisture until use, or at least drying it at 150ºC for ap.2 hours before use. Otherwise moisture can be absorbed and when exposed to rapid heating, evaporating of the absorbed water can be exstremely violent, thus completely destroying an engine-test or perhaps abort a live launch of a rocket. It is important to remember this, otherwise all previusly mentioned work can be completely wasted with one single stroke!

Some Strength Calculations.

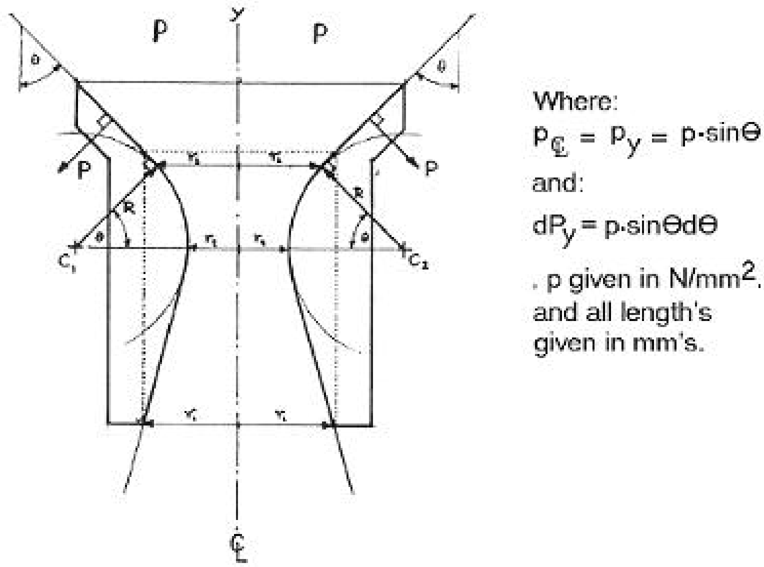
By this time it would be convenient to have a feeling about, which forces that could act on the insert and the magnitude aswell. This would give us a better idea of the survival of the insert.

The tensile strength of the sintered Alunina-ceramic material, at high temperatures are listed below in N/mm^2:

1200ºC	1400ºC	1500ºC
min. 25	18	7

Fig(VIC).

To insure ourselves that this is enough to fullfill the task we are setting, we need to know the stress a given internal peak pressure will expose the insert material to. A simple approach to this



problem will be, to divide the insert in a conically part and a torus-restricted part as shown below:

FIG(VID).

Seen from inside the chamber, the insert is supported to the radii r_1 beneath by the steel-nozzle keaper-edge. The unsupported conical part is then cairacterised by r_1 to $r_2 = R+r_t - R*\cos\theta$, and the stress contribution from this section is consequently:

$$F_c = P_{℄} \cdot A_{℄} = p \cdot \sin\theta \cdot \pi(r_1^2 - r_2^2)$$

The remaining torus-restricted surface from the angle θ to zero, is under influence of the averege pressure p_{ty}:

$$p_{ty} = \int_0^\theta dp_{ty} = \int_0^\theta p \cdot \sin\theta d\theta = p \cdot [-\cos\theta]_0^\theta = p \cdot (1 - \cos\theta)$$

, and the stress contribution from this section is then:

$$F_t = p_{ty} \cdot A_{ty} = p \cdot (1 - \cos\theta) \cdot \pi \cdot (r_2^2 - r_t^2)$$

total stress along the axis of the nozzle on the unsupported sections of the nozzle-insert, is then

$$F_{tot} = p \cdot \pi \cdot (r_2^2 \cdot (1 - \sin\theta - \cos\theta) + r_t^2(\cos\theta - 1) + r_1^2 \sin\theta)$$

Now we are able to calculate a stress in a given situation.

A Calculation Example.

To find the axial pressure on the unsupported area, we must divide the total force with the area normal to the axis:

$$p_{tot} = \frac{F_{tot}}{A_{tot}} = \frac{p\pi[(1-\cos\theta)(r_2^2 - r_t^2) + \sin\theta(r_1^2 - r_2^2)]}{\pi(r_1^2 - r_t^2)}$$
$$= \frac{p}{r_1^2 - r_t^2}[(1-\cos\theta)(r_2^2 - r_t^2) + \sin\theta(r_1^2 - r_2^2)]$$

, and now the calculated stress, on the insert, can be compared with the given figures in Fig(VIC). The pressure p must be the maximum expected peak pressure with min. factor of safety of 1½ to insure safe and reliable conditions.

$$p = p_{\max} S_f = 1\tfrac{1}{2} p_{\max}$$

To take an example, we take the case of our own manufactured insert, which have the measures:

R= 17.5 mm;	r_t =9.0 mm;	r_1 = 15.0 mm;	r_2= 14.13 mm
θ= 45°	and	p_{max}= 125 atm = 12.67 N/mm²	

P_{tot} can from these data be calculated as being:

p_{tot} = 6.96 ≈ 7 N/mm²

, meaning we exactly have a safety factor of 11 at 1500°C as seen in Fig(VIC). But if we take an example were no torus shaping has been given, meaning sharp edges between the two cones and the cylinder we get:

R= 0.0 mm;	r_1 = 15.0 mm;	r_2= r_t = 9.0 mm	θ= 45°

$$p_{tot} = (2)^{-1/2} p = \underline{\underline{13.44 N/mm^2}}$$

an increase of almost 100% of the axial stress. This shows us that smoothening of the throat in- and out-let is important.

Conclusions.

To evaluate the present results, we would encounter the fact that no actual results of static tests has been made so far. But as experiments revealed some interesting things, ideas of new experiments was born, as it often is experienced in the first phases of design and basic construction works. Our actual problems has been

a. The mixtures ratio of components.

b. Contraction bigger after heat treatment than expected.

c. To low strength and to high porousness.

d. Preventing the insert from gluing to the ovens inner surface at max.temperature of treatment.

But all of these problems are considered solved at present, and only further experiments are needed to complete a usefull prototype-insert for static testing.

APPENDIX

The used nomenclature are as follows:

Passege I. - D = The inner diameter of pressure vessel.
L = The length of the cylindrical part of vessel.
t = The wall-thickness of vessel.
p = The internal chamber pressure (N/m^2).
σ_1 = The average longitudinal stress (N/m^2).
σ_2 = The circumferential stress (N/m^2).
σ_3 = The average radial stress (N/m^2).
σ_v = The tensile yield stress of vessel-metal (N/m^2).
σ_{ref} = The desired reference tensile stress at normal working conditions (N/m^2).
S_f = The factor of safety.
z = The strength factor, due to welding effects.
p_{max} = The max.pressure for a given S_f and D,t.
t_{nom} = The nominal wall-thickness for a given S_f
dt= The tolerance of t_{nom}

II.- b = The average distance between two bolts in a row.
c = The distance from the center of a bolt to the edge of the vesselend.
d = The average bolt-hole diameter in vessel.
t = See I.
σ_v = See I.
m = The number of bolts in a row.
r = The reciprocal value of m.
d_b = The shaft diameter of bolt.
σ_b = The tensile yield strength of bolt (N/m^2).
S_f = See I.
p_{max} = See I.
p_f = One of three kinds of stress-the strength of a bolted joint are depending upon.(N).
α = Imperical value, covering effect of reduced strength in vessel,due to holes in a row.
p = The force of internal pressure acting on the cross section area of the vessel (N).

III.- $t_v = t_{vessel} = t_{nom}$ The nominal measure of wall thickness of the vessel.
θ_W = The welding gap angle.

IV.- θ_{con} = The angle of approach - convergent ½angle.
θ_{div} = The angle of exit - divegent ½angle.
A_0 = The initial approach area of nozzle.
A_t = The throat area of nozzle.
A_e = The exit area of nozzle.
d_b = See II.
D_t = The throat diameter.

VI.- Al_2O_3 = Aluminium Oxide = Corundum.
CaO = Calcium Oxide = Quick lime.
$Ca_3(AlO_3)_2$ = CalciumAluminate.
$A = Al_2O_3$
p = The internal chamber pressure (N/m^2).
p_y = The axial pressure composant (N/m^2).
θ = The angle of approach - convergent ½ angle.
r_1 = The max. radii in divergent part of insert.
r_2= The tangential radii of torus-cone contact.
r_t = The throat radii of the insert.
$A_{℄}$ = The cross section area from r_2 to r_1.

F_c = The axial force acting on $A_{℄}$ (N).
A_{ty} = The cross section area from r_t to r_2.
P_{ty} = The average axial pressure composant on the torus-restricted surface (N/m^2).
F_t = The axial force acting on A_{ty} (N)
F_{tot} = The total axial force acting on A_{tot} (N).
p_{tot} = The total axial pressure composant on the area A_{tot} (N/m^2).
A_{tot} = The total unsupported area of the insert.

<u>Usefull conversion factors to SI-units.</u>

1 atm $= 1.01325*10^5\ N/m^2 = 0.101325\ N/mm^2$
$= 1.01325*10^2$ Pa $= 0.101325$ MPa

1 kp = 9.80665 N.

List of suitable steels for pressure vessels.

Steel	DIN W. -nr.	SS (SIS)	σ_{vmin} (N/mm^2).
st. 35.8/III	1.0305	1330-15	240
st. 45.8/III	1.0405	1435-05	260
15CrMo10-II	-	2218-05	270
Stainless Steels:			
5CrNi 18-9	1.4301	2333	190
3CrNiMo 18-5	1.4417	-	450
3CrNiMo 18-13	1.4429	2375	300
5CrNiMo 18-12	1.4436	2343	210
8CrNiMo 26-5	1.4460	2324	440
17Cr 25	1.4749	2322	290

List of ISO-bolt classification of minimum strength.

ISO-class	σ_{bmin} (N/mm^2).
6.8	470
6.9	530
8.8	630
10.9	830
12.9	1060

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